EX: Find the average power in the periodic function shown in Fig. 1.

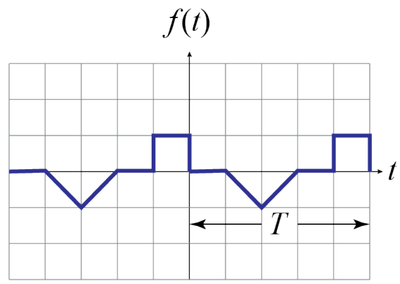

Fig. 1. Periodic signal.

SOLN: The scale of Fig. 1 is one unit per box for both axes. The period is $T = 5$. For a numerical solution, we first write a formula for one period of the function. Since the function changes its shape as it goes, we have a piecewise definition.

$$f(t) = \begin{cases} 0 & 0 < t < 1 \\ -(t-1) & 1 < t < 2 \\ t-3 & 2 < t < 3 \\ 0 & 3 < t < 4 \\ 1 & 4 < t < 5 \end{cases}$$

Note that the formulas for each sloped line segment is written as slope times x-axis intercept, i.e., $-(t-1)$.

For energy, we integrate the power, which is magnitude squared.

$$|f(t)|^2 = \begin{cases} 0 & 0 < t < 1 \\ (t-1)^2 & 1 < t < 2 \\ (t-3)^2 & 2 < t < 3 \\ 0 & 3 < t < 4 \\ 1 & 4 < t < 5 \end{cases}$$

If we write integrals for the energy calculations, we have a sum of terms, one for each nonzero interval.

$$w = \int_1^2 (t-1)^2 dt + \int_2^3 (t-3)^2 dt + \int_4^5 1 dt$$

We only need the area under each shape in the plot of magnitude squared. We may shift the shapes right or left to create simpler integrals. This is equivalent to doing a change of variables, but it is easier to do the shift visually.

We may also flip areas around the vertical axis. Only the size of the area matters. It follows that the first two integrals have the same value. So we shift the first integral one unit left and double it to account for the first two integrals. The third integral has a value of 1.

$$w = 2\int_0^1 t^2 dt + 1$$

or

$$w = 2 \cdot \frac{1^3}{3} + 1 = \frac{5}{3}$$

We now have the total energy in one period. Dividing by the period, $T = 5$, gives the average power in the periodic signal.

$$p = \frac{5/3}{5} = \frac{1}{3}.$$