Ex: Given $x(t)$ shown in Fig. 1, sketch $\dfrac{dx(t)}{dt}$ using the blank axes below in Fig. 2.

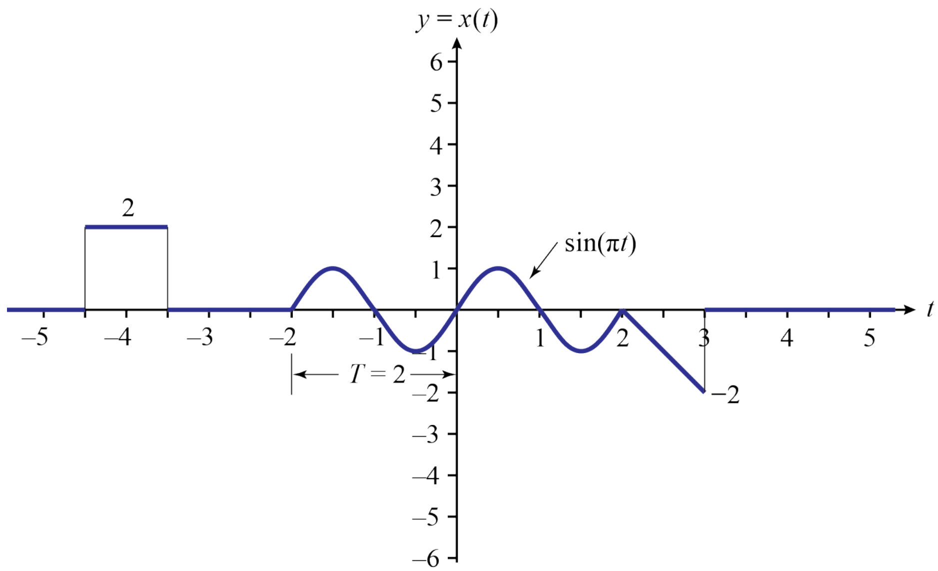

Fig. 1. Example function.

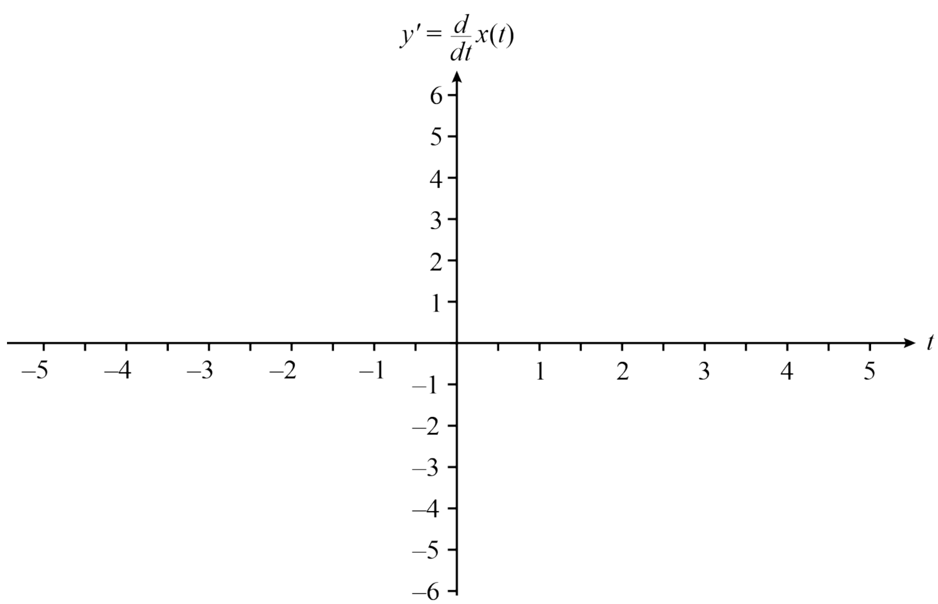

Fig. 2. Blank axes for plotting derivative of $x(t)$.

SOLN: Fig. 3 shows the derivative of $x(t)$, which steps up by two units at −4.5 and steps down by two units at −3.5. The derivative of a unit step is an impulse, $\delta(t)$. Thus, we get impulses (shown as lollipops) of area 2 and −2 for the rectangular pulse between −4.5 and −3.5 in $x(t)$.

The derivative of $\sin(\pi t)$ is $\pi\cos(\pi t)$ using the chain rule. The derivative steps up by π at $t = -2$, since the derivative suddenly changes at $t = -2$.

The derivative changes suddenly again at $t = 2$. The downward slope (i.e., derivative) at $t = 2$ is $\dfrac{\Delta y}{\Delta t} = \dfrac{-2}{1} = -2$. The slope is constant up to t=3, after which x(t) steps up by two units, causing another impulse function (of area 2) in the derivative.

Elsewhere, $x(t)$ is horizontal with a derivative of zero. Note that a derivative may zero for horizontal segments that are nonzero.

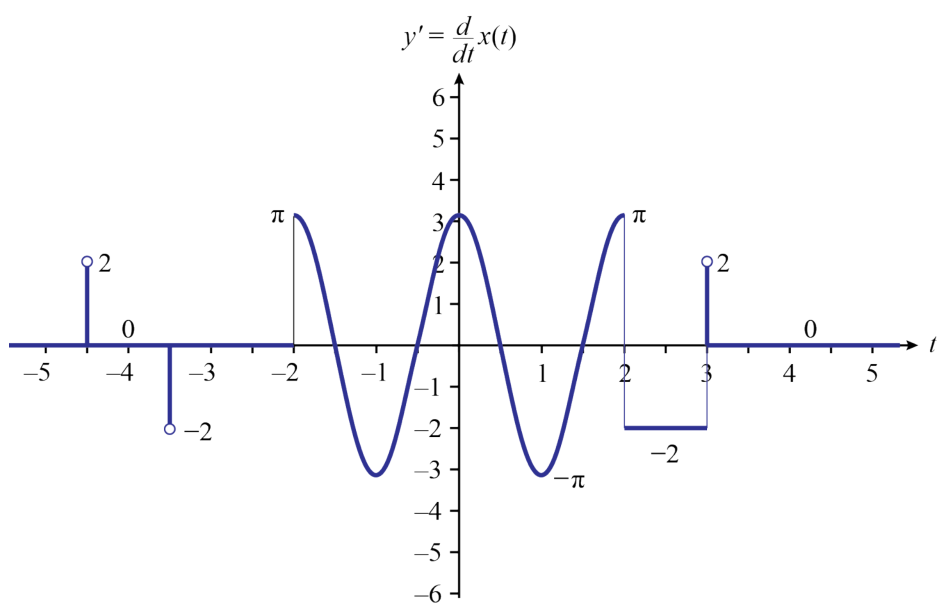

Fig. 3. Derivative of $x(t)$.