EX: Fig. 1 shows a function, $x(t)$. The generic form of a linear function transformation of this function is $ax(b[t-c])$. Answer the questions below about transformations of $x(t)$.

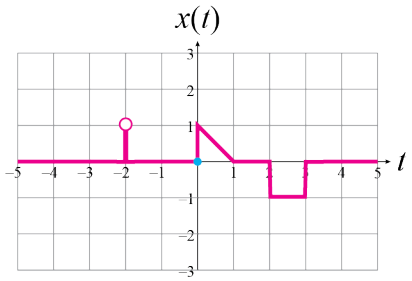
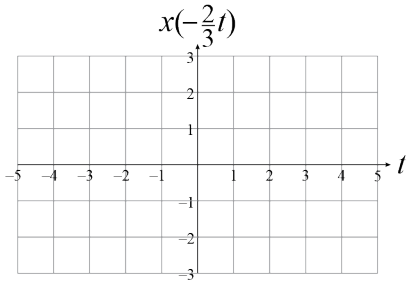
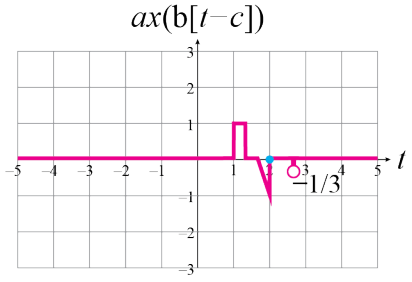


Fig. 1. Function being transformed.

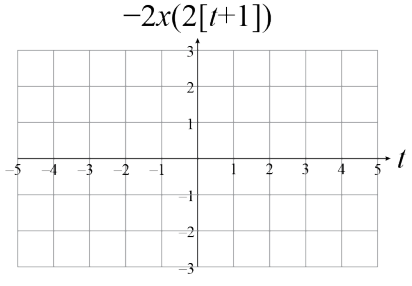
a. Draw a graph of $x\left(-\frac{2}{3}t\right)$ on the axes provided.



b. Find the value of a, b, and c for the linear transformation shown.

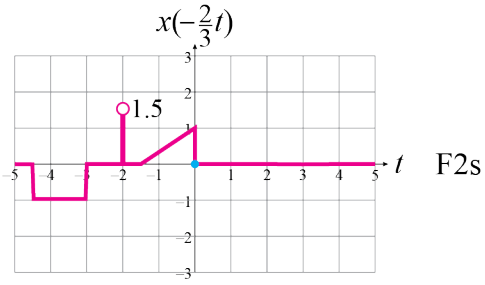


c. Draw a graph of $-2x(-2[t+1])$ on the axes provided.



SOLN: a) The parameters of this transformation are $a = 1$, $b = -\frac{2}{3}$, and $c = 0$. Time is reversed and moving only 2/3 as fast, meaning $x(t)$ gets stretched out by a factor of 3/2 (the reciprocal of 2/3). The origin stays put at $t = 0$ and the scale of the function vertically is unchanged.

Why is the area of the impulse function different from before? This change in the area of the impulse is a result of the identity for scaling of impulses: $\delta(bt) = \frac{1}{|b|}\delta(t)$. This curious behavior of impulses arises from their property of being an area rather than a height. Mathematically, the area gets larger if $|b| < 1$. Here, since $b = -\frac{2}{3}$, the impulse is scaled by $\frac{3}{2}$.



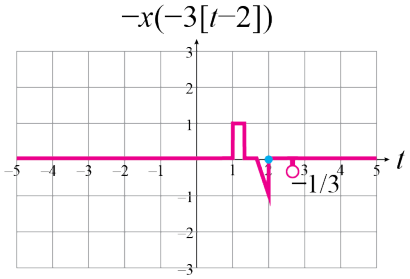
F2s

b) The origin, denoted by a blue dot, has moved to $t = 2$, meaning $c = 2$.

Time is reversed and moving 3 times as fast, based on the shapes in $x(t)$ between 0 and 3 being reversed and compressed into an interval of one time unit. Thus, $b = -3$.

The scale of the function vertically is unchanged except that it is upside-down. Thus $a = -1$.

The impulse function area changes according to the identity for scaling of impulses: $\delta(bt) = \frac{1}{|b|}\delta(t)$. Thus, if $b = -3$, the area of the impulse is scaled by $\frac{1}{3}$.



c) The transformed function, relative to $x(t)$, is shifted one time unit to the left ($c = -1$ from $t + 1 = t - -1$), compressed in time by a factor of two ($b = 2$), and scaled up by a factor of 2 and inverted ($a = -2$).

The impulse gets scaled by $\frac{a}{|b|} = \frac{-2}{|2|} = -1$. Note that a acts outside the impulse function and scales the impulse as expected.