Ex: Fig. 1 shows a function, $x(t)$. The generic form of a linear function transformation of this function is $ax(b[t-c])$. Answer the questions below about transformations of $x(t)$.

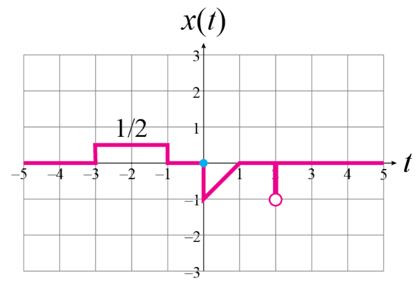
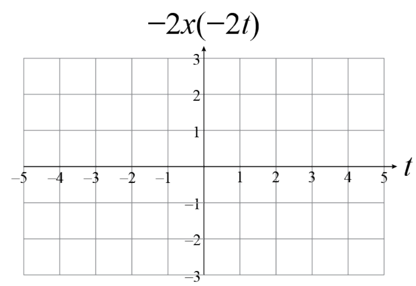
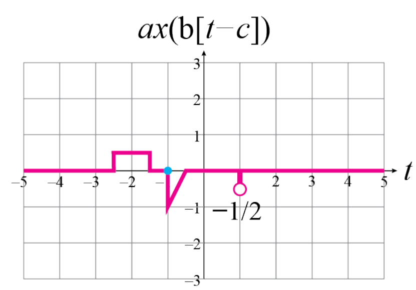


Fig. 1. Function being transformed.

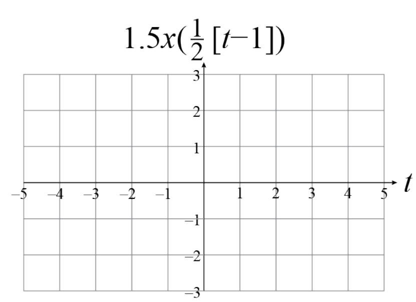
a. Draw a graph of $-2x(-2t)$ on the axes provided.



b. Find the value of a, b, and c for the linear transformation shown.

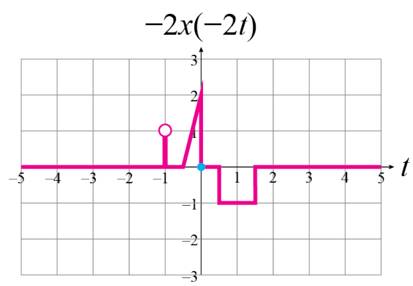


c. Draw a graph of $1.5x\left(-\frac{1}{2}[t-1]\right)$ on the axes provided.



SOLN: a) The parameters of this transformation are $a = -2$, $b = -2$, and $c = 0$. Since $b = -2$, time is reversed and moving only twice as fast, meaning $x(t)$ gets compressed by a factor of 2. The origin stays put at $t = 0$ and the scale of the function vertically is unchanged.

The impulse gets scaled by $\dfrac{a}{|b|} = \dfrac{-2}{|-2|} = -1$. Note that a acts outside the impulse function and scales the impulse as expected.

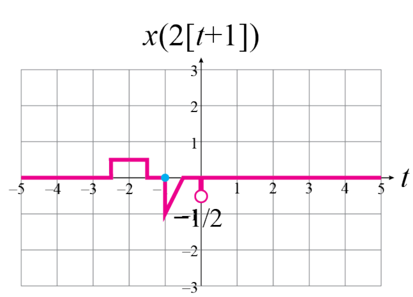


b) The origin, denoted by a blue dot, has moved to $t = -1$, meaning $c = -1$.

Time is compressed by a factor of two, based on the shapes in $x(t)$. Thus, $b = 2$.

The scale of the function vertically is unchanged. Thus $a = 1$.

The impulse gets scaled by $\dfrac{a}{|b|} = \dfrac{1}{|2|} = \dfrac{1}{2}$. Since the impulse area started out as -1, the new area is $-\frac{1}{2}$.



c) The transformed function, relative to $x(t)$, is shifted one time unit to the right ($c = 1$). The function is expanded in time by a factor of two ($b = \frac{1}{2}$), and scaled up by a factor of 1.5 ($a = 1.5$).

The impulse gets scaled by $\dfrac{a}{|b|} = \dfrac{1.5}{|1/2|} = 3$.