TOOL: The rules below describe how to graph linear transformations of a function, $x(t)$.

The generic form of a linear function transformation is $y(t) = ax(b[t-c])$.

Fig. 1 shows an example transformation, $y(t) = -2x(-1.5t - 3)$ where $x(t)$ is the sawtooth shape shown:

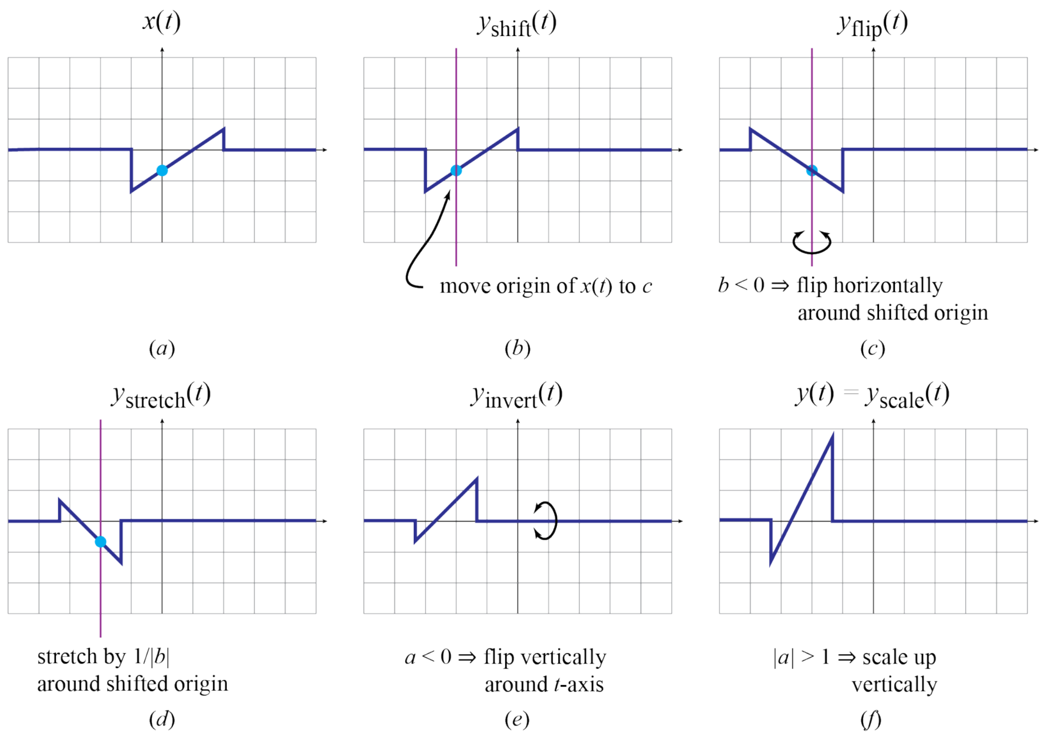

Fig. 1. Function and transformation steps.

1. The first step in graphing $y(t)$ is to put the argument transformation in proper form.
   - Write the argument in the form $b[t-c]$.

   Here, the argument for $y(t)$ is $-1.5t - 3 = -1.5[t-(-2)]$. So our transformed function is written as

   $$y(t) = ax(b[t-c]) = -2x(-1.5[t+2]) = -2x(-1.5[t-(-2)]).$$

   We have the following parameters:

   $a = -2 \qquad b = -1.5 \qquad c = -2$

This form of the argument shows that, when $t = c$, the value of the argument of $y(t)$ will be zero. That is, $y(c) = x(0)$, meaning we shift the copy of $x(t)$ sideways to put its origin at c.

This gives us the graph of $y_{\text{shift}}(t)$ shown in Fig. 1b. Note that a positive value of c moves $y(t)$ to the *left*, and a negative value of c moves $y(t)$ to the *right*.

2. The second step in graphing $y(t)$ is to decide whether to flip the copy of $x(t)$ horizontally. The sign of b determines whether we flip the function horizontally.

    - If (and only if) $b < 0$, flip the copy of $x(t)$ around a vertical line at c.

    Here we flip the copy of $x(t)$ horizontally since b is negative. See Fig. 1c. If b is positive, no flipping.

3. The third step in graphing $y(t)$ is to stretch or compress the copy of $x(t)$ horizontally. The value of $|b|$ determines the value of stretch or compression.

    A value of $b < 1$ will cause $y(t)$ to be an expanded copy of $x(t)$ around c. A value of $b > 1$ will cause $y(t)$ to be a contracted copy of $x(t)$ around c.

    To see why the copy is expanded or contracted, consider how t must change for the value of the argument, $|b|[t-c]$, to change by one time unit. That is, we consider $|b|t = 1$, which corresponds to a change of one time unit for the graph of $x(t)$. So the value of $y(t = c + \frac{1}{|b|}) = x(1)$.

    - The factor of expansion or contraction is $1/|b|$ around c.

    Here we contract the copy of $x(t)$ by $1/1.5 = 2/3$. See Fig. 1d.

4. The fourth step in graphing $y(t)$ is to flip vertically if (and only if) $a < 0$.

    - Flip the graph of $y(t)$ vertically around t-axis if $a < 0$.

    Here we flip the copy of $x(t)$ since a is negative. See Fig. 1e.

5. The fifth and final step in graphing $y(t)$ is to scale vertically. The value of $|a|$ causes a vertical stretch or contraction of $y(t)$. This stretch is like grabbing the handle of a photo to make it taller or shorter in Photoshop. In contrast to the direction of movement being counterintuitive, the image, $y(t)$, becomes taller if $|a| > 1$ and shorter if $|a| < 1$.

- The plot of $y(t)$ is scaled by $|a|$.

Here we scale vertically by 2 since $|a| = 2 > 0$. See Fig. 1f. Done.