EX: Use the rules for areas under a quadratic (or parabolic) function to find the area under g(t) shown below.

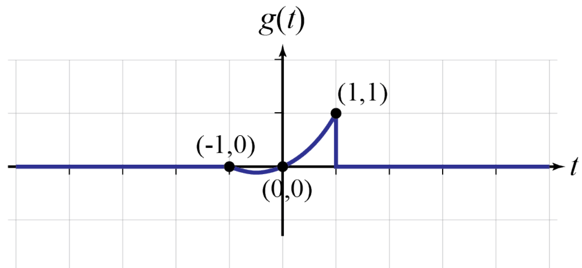
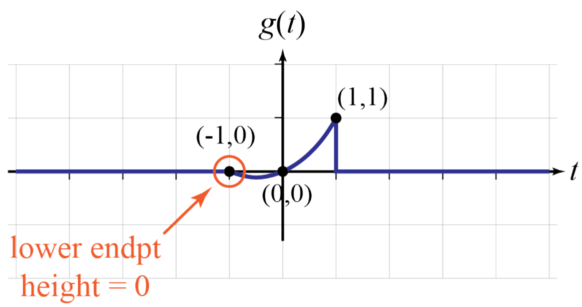


Fig. 1. Caption.

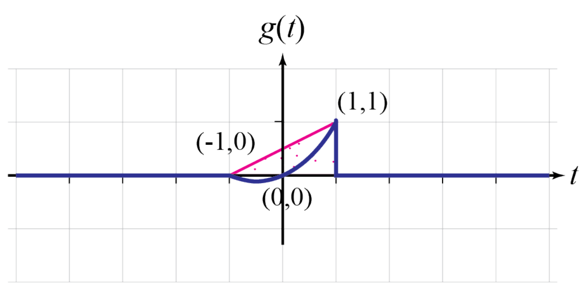
SOLN: Rectangle area



We use the lower of the endpoints, which means we use $y_w = g(t = -1) = 0$ at the left end of the parabolic. The width of the area is $L = 2$, so the area for the rectangular part of our answer is

$$A_{\text{rect}} = L y_w = 2 \cdot 0 = 0$$

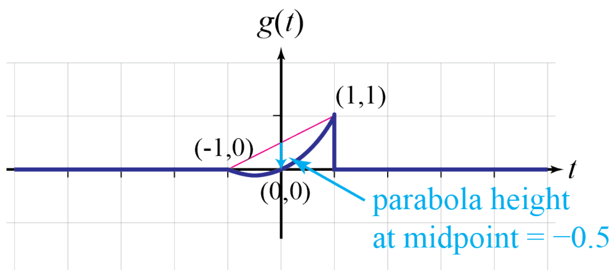
Triangle area



There is no rectangular base value to subtract, so we proceed to attach the endpoints of $g(t)$ and find the area of that triangle. The triangle goes from (-1,0) to (1,1). The change in height from one end to the other is $y_t = 1 - 0 = 1$. The width, which remains the same throughout the calculation, is $L = 2$. We use the formula for triangle area.

$$A_{tri} = \frac{1}{2} L y_t = \frac{1}{2} \cdot 2 \cdot 1 = 1$$

<u>Quadratic area</u>



To find the height of the parabola (quadratic part), we go to the midpoint of the area horizontally. The midpoint of the area here is at $t = 0$. The height of the triangle at $t = 0$ in the previous step is 0.5. The height of the function is $g(0) = 0$. The difference between the height of the function and the triangle at $t = 0$ is

$$y_p = 0 - 0.5 = -0.5.$$

The parabolic area is

$$A_{parab} = \frac{2}{3} L y_p = \frac{2}{3} \cdot 2 \cdot -0.5 = -\frac{2}{3}.$$

<u>Total Area</u>

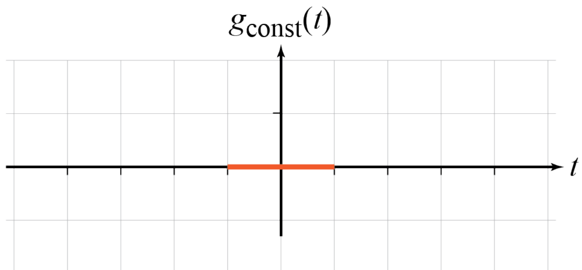
$$A = A_{rect} + A_{tri} + A_{parab} = 0 + 1 - \frac{2}{3} = \frac{1}{3}$$

Note that the area should be positive by inspection, since there is more area above the horizontal axis than below it. Given the curvature of the function between 0 and 1, the area should be a bit less than 1/2, which is the area of a triangle from (0,0) to (1,1). Our answer of 1/3 seems reasonable. If we were to make a visual estimate of the area, we would come up with something close to 1/3.

Taking a closer look (which is unnecessary for the area calculation but is presented here for completeness), we have the following breakdown of the original curve into its component parts.

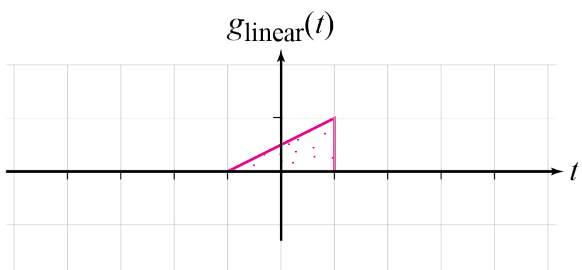
Constant offset is area = 0.

$$g_{\text{const}} = 0$$



Linear (sloped) part is a triangle of area = 1.

$$g_{\text{linear}} = \begin{cases} 0 & t < 0 \\ \frac{1}{2}(t-1) & -1 \le t \le 1 \\ 0 & 1 < t \end{cases}$$



Quadratic part is parabola of area = 2/3.

$$g_{\text{linear}} = \begin{cases} 0 & t < 0 \\ \frac{1}{2}(t-1)(t+1) & -1 \le t \le 1 \\ 0 & 1 < t \end{cases} \qquad \text{roots at } \pm 1, \text{ ht } \frac{1}{2} \text{ at } 0$$