EX: Compute $\int_{-\infty}^{t} f(t)dt$ graphically for $f(t)$ shown in Fig. 1.

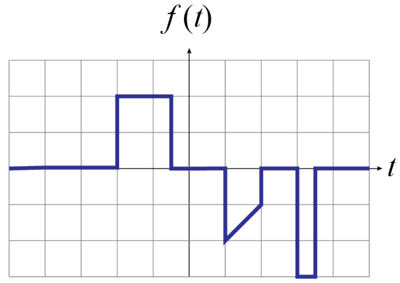

Fig. 1. Function $f(t)$.

SOLN: Fig. 2 shows the geometric shapes we use to graph the integral. For each geometric shape, the change in the integral over the width of the area will be equal to the area. That is, for the first shape, the area is 3, so the integral of $f(t)$ must change by +3 over the interval from $t = -3$ to $t = -1.5$. Also, the height of the area at a given point is the slope of the integral curve. Fig. 3 shows the integral curve with annotations for areas and slopes used to graph the integral.

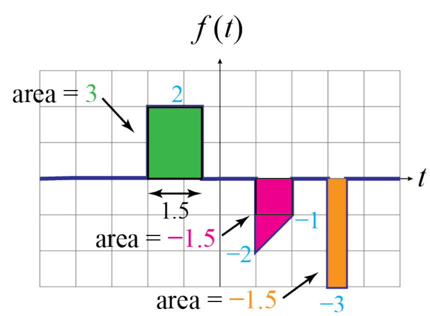

Fig. 2. Areas for function $f(t)$.

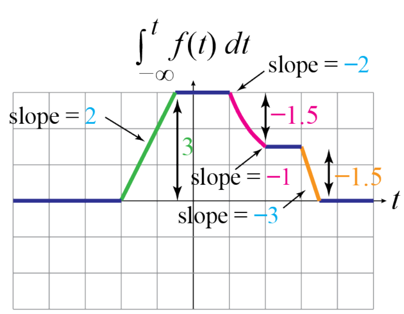

Fig. 3. Integral of $f(t)$ based on areas of geometric shapes and slopes.

Where $f(t) = 0$ between geometric shapes, the value of the integral remains constant.

For the second area from $t = 1$ to $t = 2$, we note that it is a square plus a triangle, the total area of which is -1.5, which is the change in the integral over the interval where the second area is located.

The integral of the sloped line in the second area must be a quadratic function. The slope of the integral at the start and end of the interval for the second area are the values of $f(t)$ at the ends of the interval. This information helps us make a more accurate sketch of the quadratic section.

The last area is narrow and negative and rectangular. The value of the integral must drop by the area of the rectangle, which is -1.5, in the interval from $t = 3$ to $t = 3.5$.

NOTE: The value of the integral in this example is zero on the right side of the graph. An integral may be nonzero as $t \to \infty$ in general, but in many problems such as convolutions there may be an obvious reason to expect the value of the integral to be zero as $t \to \infty$, which is useful constraint for spotting errors in the sketch of the integral.