EX: Compute $\int_{-\infty}^{t} f(t)dt$ graphically for $f(t)$ shown in Fig. 1. This $f(t)$ is similar to what might occur when computing a convolution. The lollipops represent impulse functions. The height of a lollipop represents the area of the impulse.

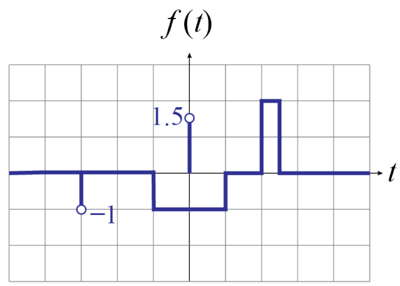

Fig. 1. Function $f(t)$.

SOLN: Fig. 2 shows the geometric shapes we use to graph the integral. When an integral passes an impulse, the value of the integral steps up or down by the area of the impulse.

For each rectangle, the change in the integral over the width of the area will be equal to the area. That is, for the first shape, the area is −2, so the integral of $f(t)$ must change by −2 over the interval from $t = -1$ to $t = 1$.

Also, the height of the area at a given point in $f(t)$ is the slope of the integral curve. If $f(t)$ is constant, the slope of the integral is constant. Fig. 3 shows the integral curve with annotations for areas and slopes used to graph the integral.

Where $f(t) = 0$ between geometric shapes, the value of the integral remains constant.

In this example, the second impulse occurs in the middle of the rectangular area, causing a step up, after which the integral continues to fall because of the rectangular area. In convolution integrals, an impulse may occur at a side of a rectangle. The impulse may be difficult to see, so close inspection is prudent.

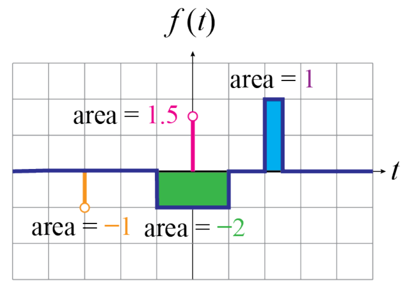

Fig. 2. Areas for function $f(t)$.

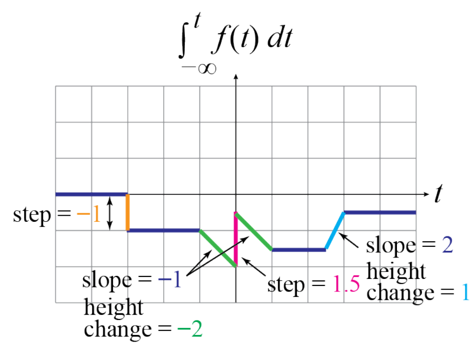

Fig. 3. Integral of $f(t)$ based on areas of impulses and rectangles, and values of slopes.

NOTE: The value of the integral in this example is nonzero on the right side of the graph. The value of the integral will remain constant as $t \to \infty$.