TOOL: The following approach enables the rapid calculation of areas under curves (i.e., definite integrals) for piecewise segments that are constant, linear, or quadratic. The idea is to represent the segement of the curve as a sum of a constant (area of rectange) plus a sloped line (area of triangle) plus a quadratic that is zero at both ends (area of symmetrical parabola). Let the horizontal length of the curve segment be L.

1. If both ends of the curve segment are at height zero, bypass Step 1. Otherwise, draw a rectangle that corresponds to the lower of the two end points. Let y_w be the lower of the two endpoints. The formula for the area of the rectangle is length times height.

   $$A = Ly_w$$

   Calculate the area of the rectangle and then remove the rectangle to lower (or raise) the curve segment so one end starts at height zero.

2. If both ends of the remaining curve segment are at height zero, bypass Step 2. Otherwise, draw a straight line from the left end of the remaining curve to the right end of the curve, forming a triangle with its base on the horizontal axis. Let y_t be the lower of the two endpoints. The formula for the area of the triangle is one-half length times height.

   $$A = \frac{1}{2}Ly_t$$

   Calculate the area of the triangle and then subtract the triangle from the segment so both endpoints are at height zero.

3. If the remaining curve segment is zero for its entire length, bypass Step 3. Otherwise, determine the height of the center point of the remaining parabolic curve. Let y_p be the height of the center point. The formula for the area of the parbolic curve is two-thirds length times height:

   $$A = \frac{2}{3}Ly_p$$

   Calculate the area of the parabolic curve and then subtract the area from the remaining segment to get zero for the entire remaining segment.

4. The sum of the areas of the rectangle, triangle, and parabola is the area under the segment of the original curve.

Note: The above process may be carried out without actually subtracting the curves at each step.

The figure, below, shows examples of area (integral) calculations.