

TOOL: The following rules govern the graphing of $F(t) = \int_{-\infty}^t f(t)dt$ given the shape of $f(t)$ in a given interval, $[t_1, t_2]$, from time t_1 to t_2 .

1. The height of $F(t_1)$ at the start of the interval is equal to the area, A , under the curve up to time t_1 . That is, $F(t_1) = A$ at the start of the interval. If necessary, area A may be approximated by visual inspection.
2. The increase in height of $F(t)$ at the end of the interval is equal to the area under $f(t)$ between t_1 and t_2 . If the shape of $f(t)$ in $[t_1, t_2]$ is a simple geometric shape such as a rectangle or triangle, the increase in height of $F(t)$ in $[t_1, t_2]$ may be computed from formulas, such width-times-height for rectangles or one-half base-times-height for triangles.
3. The slope of $F(t)$ at the start of the interval is equal to the value of the function, $f(t_1)$, and the slope of $F(t)$ at the end of the interval is equal to the value of the function, $f(t_2)$. The slopes may be viewed as tangent lines at t_1 and t_2 for sketching $F(t)$, similar to the way Bezier curves are created by clicking at points and dragging in the direction of the tangent line in drawing programs.
4. How quickly $F(t)$ veers away from a tangent line depends on the second derivative of $F(t)$, i.e., the curvature, which is the first derivative of $f(t)$. If the magnitude of the derivative of $f(t)$ is high, $F(t)$ veers away from its tangent line more quickly.

For simple shapes of $f(t)$, the following graphing rules apply. Note that any $f(t)$ may be approximated by these simple shapes.

- a) If $f(t)$ is zero in $[t_1, t_2]$, the height of $F(t)$ remains constant at height A on interval $[t_1, t_2]$.
- b) If $f(t)$ is constant in $[t_1, t_2]$, the height of $F(t)$ is a straight line on $[t_1, t_2]$ whose slope is equal to the value of the constant.
- c) If $f(t)$ is a line segment in $[t_1, t_2]$, the height of $F(t)$ is a quadratic function on $[t_1, t_2]$ with initial slope at t_1 equal to $f(t_1)$ and final slope at t_2 equal to $f(t_2)$.

The figure, below, shows the application of the integral graphing rules to $f(t)$ (top) to obtain the integral (bottom).

