

TOOL: The following procedure creates an accurate sketch of the product of a sinusoid and a linear functions of arbitrary slope. Figs. 3-7 show an example with the step numbers circled.

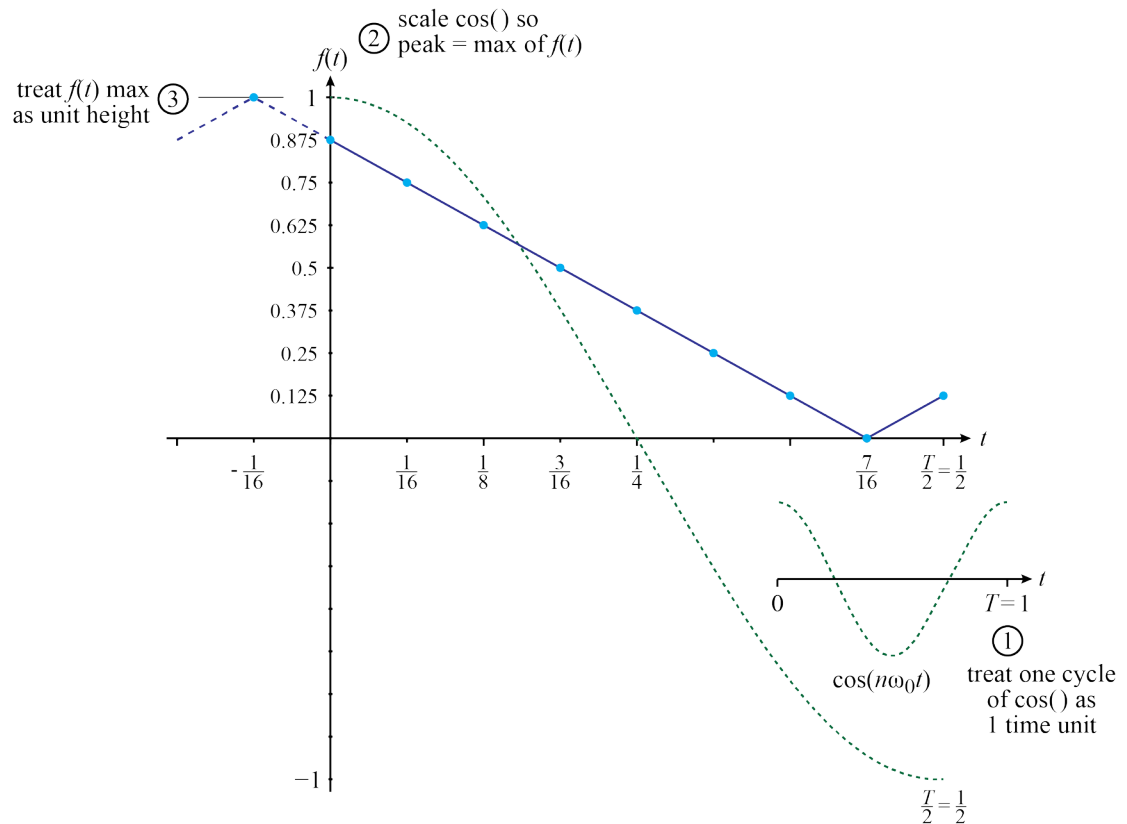


Fig. 1. First three steps in calculating area of slanted line times cosine are to normalize scaling.

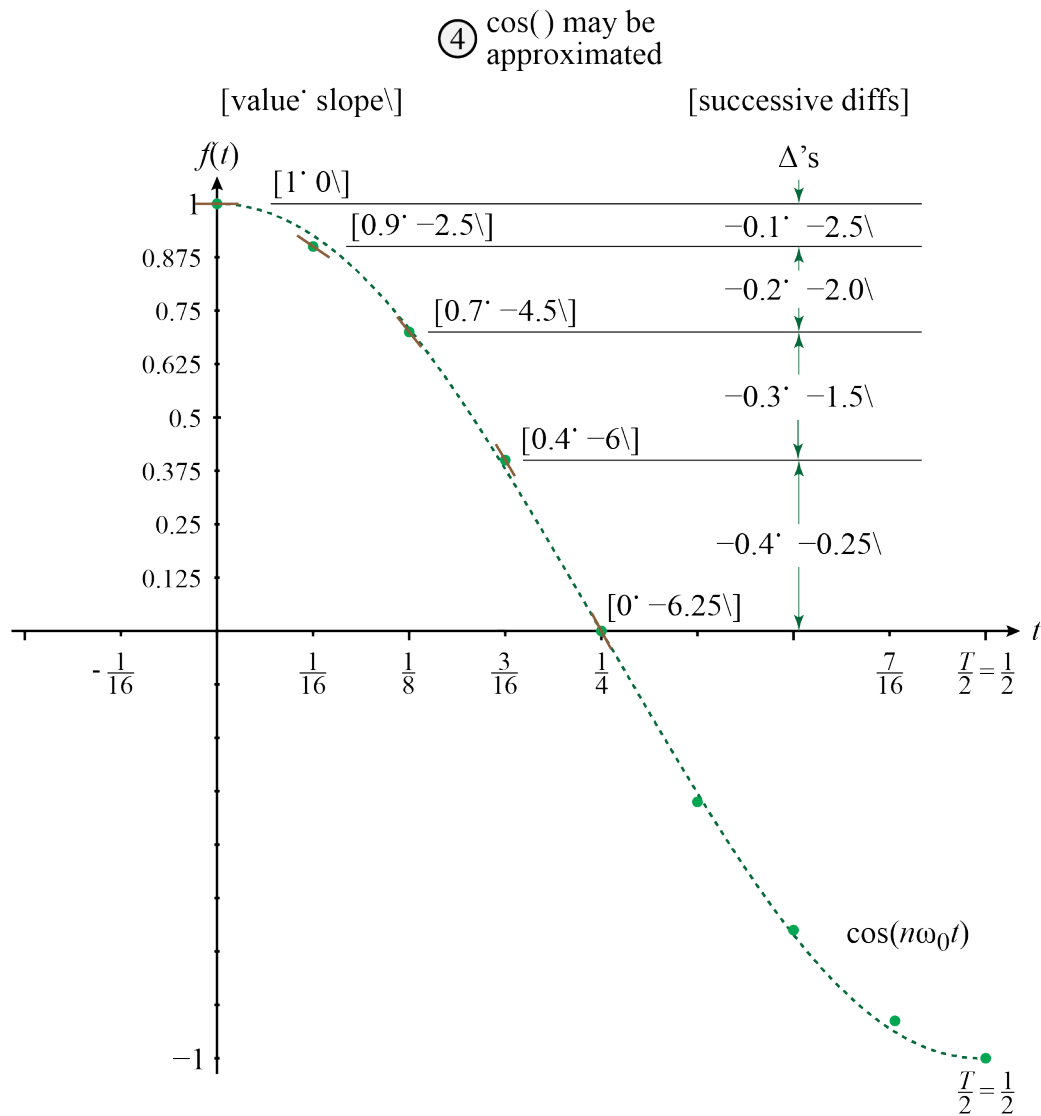


Fig. 2. Fourth step in calculating area of slanted line times cosine is to approximate the cosine function (but only used if a detailed drawing is desired). The cosine function may be approximated by successive drops from unity that increase by 0.1 at each $1/16$ of a cycle of the cosine. The slopes may similarly be approximated by drops from zero that start at -2.5 and decrease by 0.5 at each $1/16$ of a cycle of the cosine until the last step that drops only 0.25 from the preceding slope.

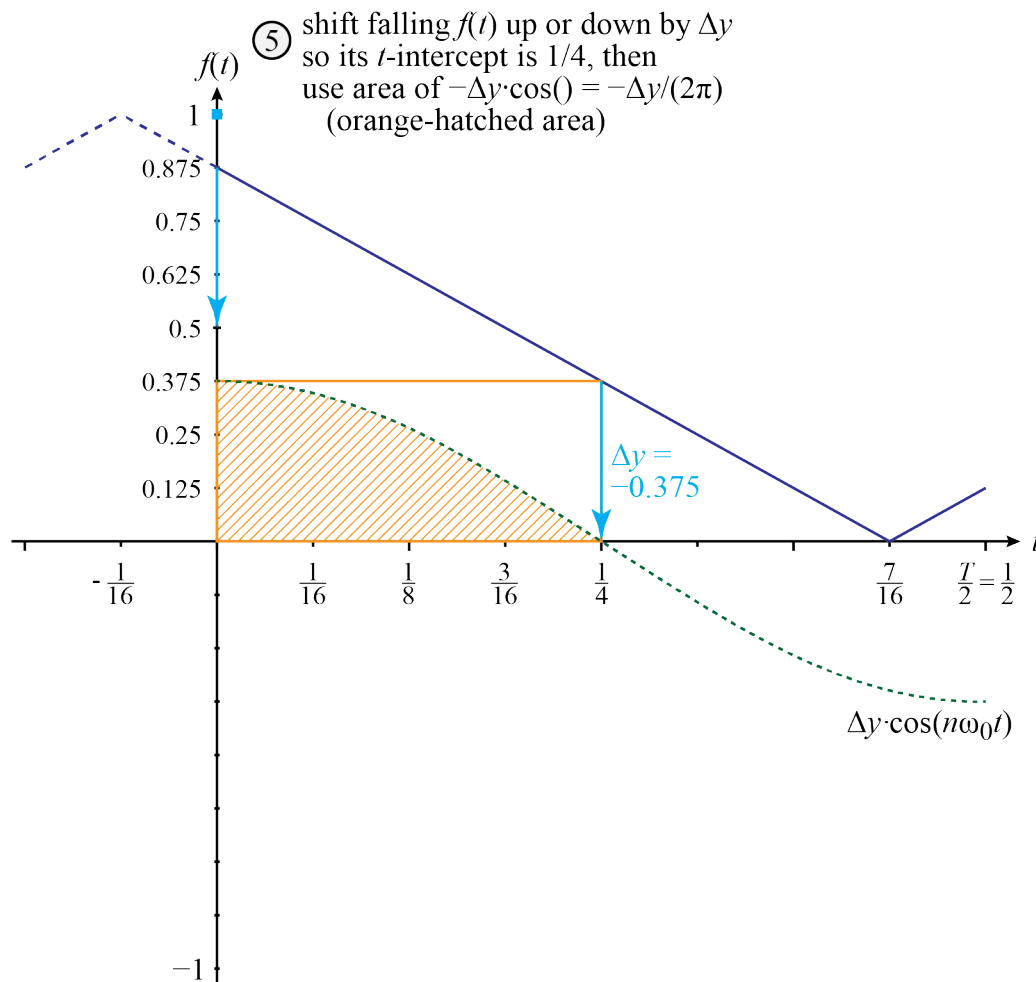


Fig. 3. Fifth step in calculating area of slanted line times cosine is to treat the line as a constant plus a slanted line that crosses through zero at $1/4$ cycle. The area of the constant times the cosine is the negative of the vertical shift times $1/(2\pi)$.

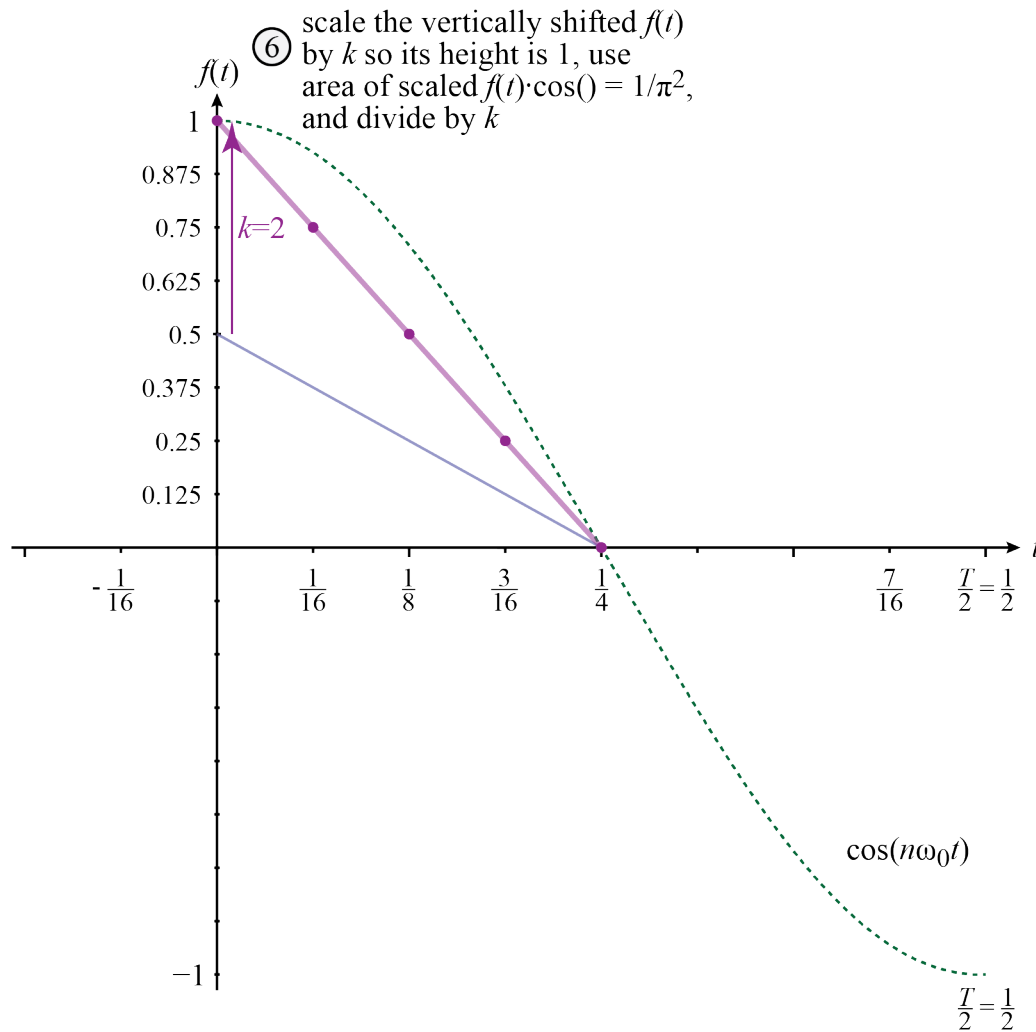


Fig. 4. Sixth steps in calculating area of slanted line times cosine is to use the standard area of line of unit height at zero times the cosine being $1/\pi^2$ in one fourth of a cycle of the cosine.

The next three steps are elaborations on the calculations of the areas of a constant and a standard slanted line times 1/4 cycle of a cosine.

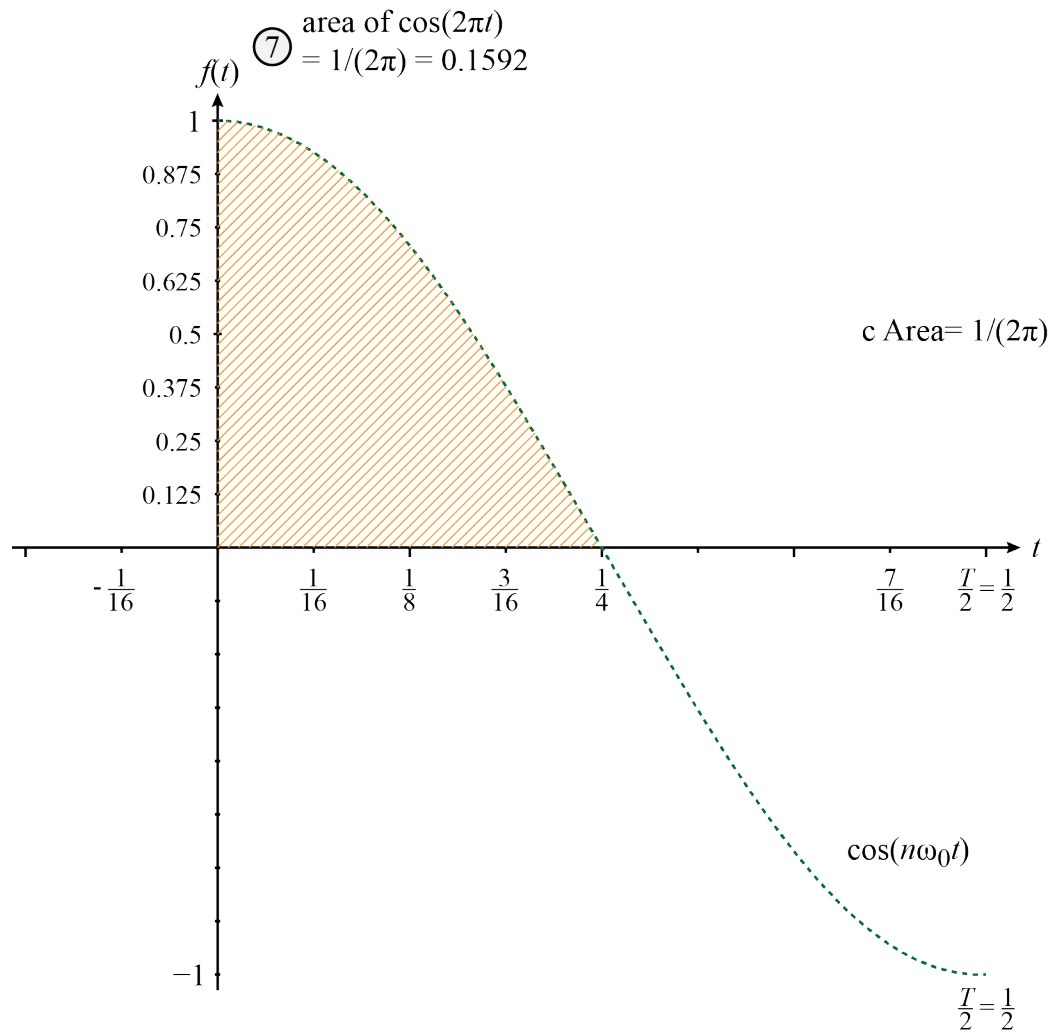


Fig. 5. Seventh step in calculating area of slanted line times cosine is a detailed look at the area of a cosine times unity for 1/4 cycle. The letter 'c' may be used to denote areas of this shape on a sketch of the graphical calculations.

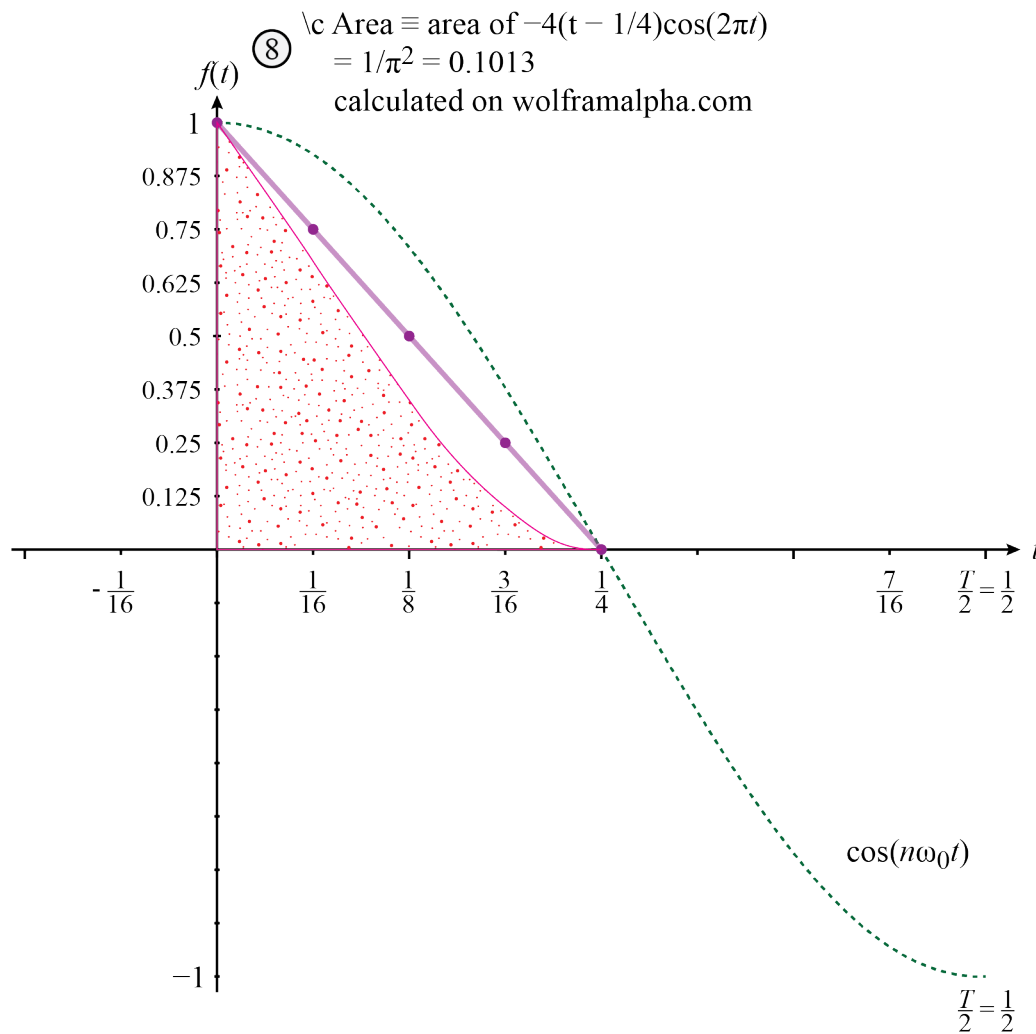


Fig. 6. Eighth step in calculating area of slanted line times cosine is a detailed look at the area of a cosine times a standard downward slanted line for 1/4 cycle. The designation '\c' may be used to denote areas of this shape on a sketch of the graphical calculations.

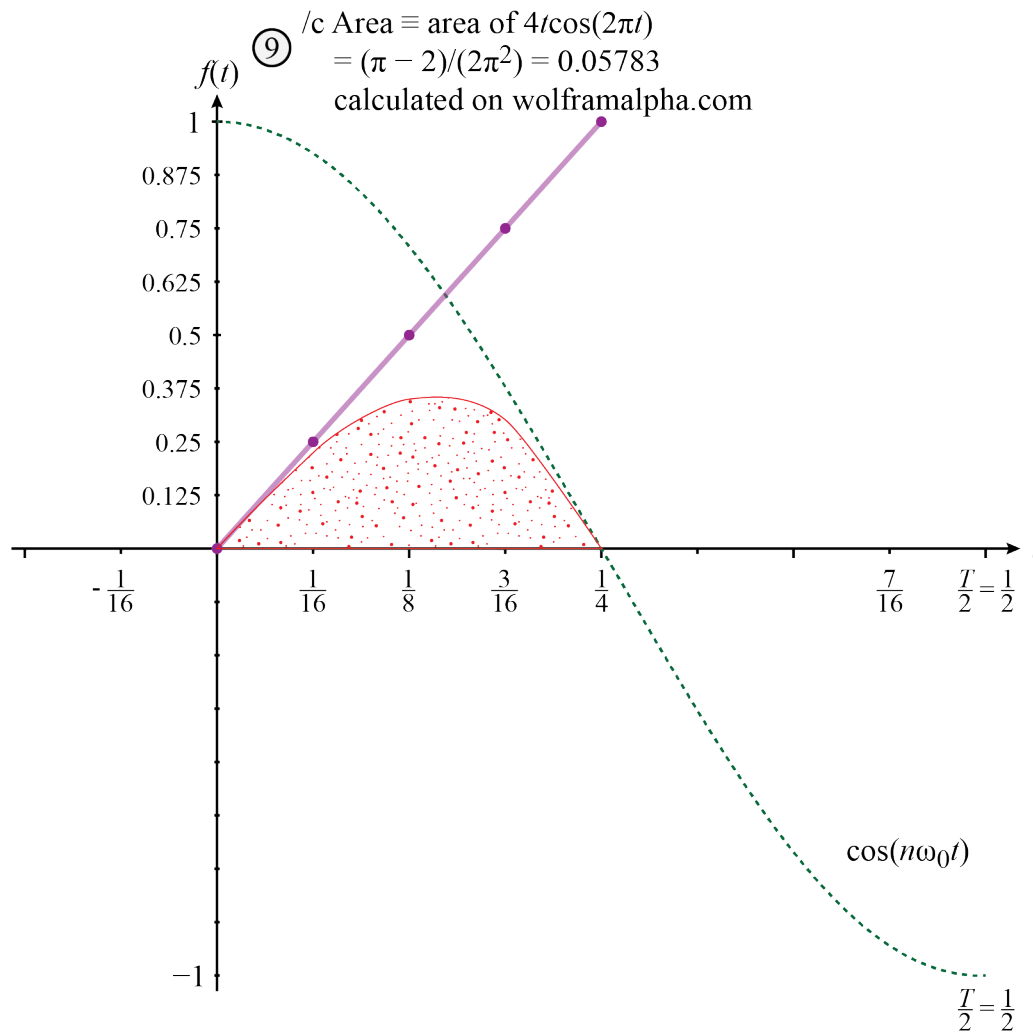


Fig. 7. Ninth step in calculating area of slanted line times cosine is a detailed look at the area of a cosine times a standard upward slanted line for 1/4 cycle. The designation '/c' may be used to denote areas of this shape on a sketch of the graphical calculations.

Note: the area of the downward and upward slanted lines times the cosine, if summed, would give the same area as unity times the cosine. Thus, the area in Fig. 7 may be derived from the areas in Figs. 5 and 6.

$$\frac{1}{2\pi} - \frac{1}{\pi^2} = \frac{\pi - 2}{2\pi^2}$$