

TOOL: By expressing a trapezoid as a constant plus a slanted line intersecting the horizontal (time) axis at a convenient value, we may use the following integral formulas to calculate the area under a trapezoid times a cosine waveform:

$$\int_0^{\tau} \cos(at) = \frac{\sin(at)}{a} \Big|_0^{\tau} = \frac{\sin(a\tau)}{a}$$

$$\int_0^{\tau} t \cos(at) = \frac{at \sin(at) + \cos(at)}{a^2} \Big|_0^{\tau} = \frac{a\tau \sin(a\tau) + \cos(a\tau) - 1}{a^2}$$

ref:

[https://www.wolframalpha.com/input?
i2d=true&i=Integrate%5Bx+cos%5C%2840%29xa%5C%2841%29%2Cx%5D](https://www.wolframalpha.com/input?i2d=true&i=Integrate%5Bx+cos%5C%2840%29xa%5C%2841%29%2Cx%5D)