IMPULSE OR DELTA FUNCTION
DEFINITION

TOOL: An impulse function, $\delta(t)$, (or delta function) is zero everywhere but at $t = 0$. At $t = 0$, the value of the impulse function is infinity. The area of the impulse function is equal to one.

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \\ \int_{-\infty}^{\infty} \delta(t)dt = 1 & \end{cases}$$

The impulse function may be thought of as a point mass in a probability density function or as a value of area concentrated at a single point.

The impulse function is the limit of rectangle functions (or triangle functions, or any of a number of other continuous functions) of unit area that get narrower and taller (while maintaining a unit area) at each step.

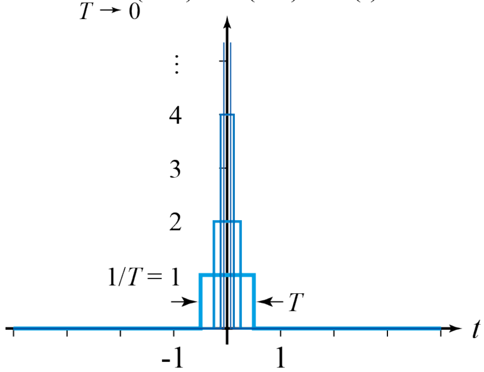
$$\delta(t) = \lim_{T \to 0} \left(\frac{1}{T}\right) \text{rect}\left(\frac{t}{T}\right)$$

where

$$\text{rect}(t) = \begin{cases} 0 & |t| > 1/2 \\ 1 & |t| = 1 \end{cases}$$

Graphically, we have rectangle functions with a constant area of unity getting narrower and taller:

$$\lim_{T \to 0} (1/T)\text{rect}(t/T) = \delta(t)$$



The impulse function may be thought of as a point mass in a probability density function or as a value of area concentrated at a single point.

TOOL: When an impulse function is multiplied by a constant, the area of the impulse function is scaled by the constant:

$$k\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \\ \int_{-\infty}^{\infty} \delta(t)dt = k & \end{cases}$$

Note that the width of the scaled impulse function is still zero and the height is still infinity.

NOT'N: The impulse function is usually drawn as a bold vertical lollipop or arrow whose height indicates the area under the impulse function. The value of the area may also be written next to the lollipop or arrow if a precise value is of interest.