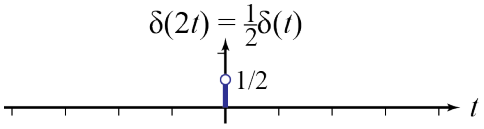
TOOL: Unlike most functions that become narrower or wider when the time variable is scaled, the impulse function changes its area. Thus, time scaling is equivalent to amplitude scaling.

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

For example, $\delta(2t) = \frac{1}{2}\delta(t)$:



PROOF: Treat the time-scaled impulse function as the limit of time scaled rectangle functions. Each rectangle has a area that scales inversely with the time scaling. The area stays constant at the scaled value as the limit becomes a scaled impulse function.