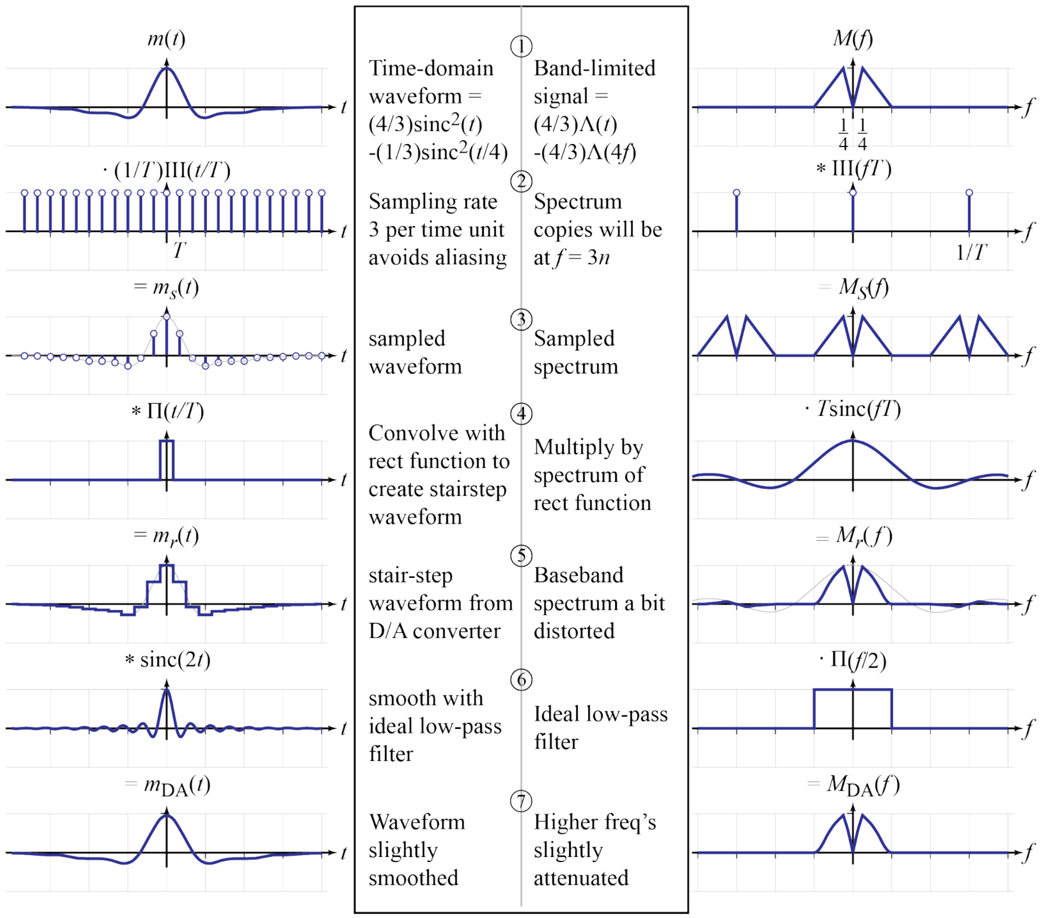
TOOL: The Rosetta Stone method reveals the effects of Analog-to-Digital and Digital-to-Analog conversion. The diagram below shows the analysis, which is followed by a line-by-line description.



Line 1 The process begins with a waveform such as an audio signal. We use a generic signal with a spectrum similar in shape to audio but strictly band-limited to eliminate any issue of aliasing.

In practice, the next step might be a low-pass filter that would slightly distort the spectrum. However, a high-quality filter will have a minimal effect on the waveform, so we omit that filtering step to focus on other issues that arise later in the process.

Line 3 The waveform is sampled, typically by a sample-and-hold circuit followed by some sort of Analog-to-Digital (A/D) converter circuit. We model the sampling process as an impulse train (Line 2) multiplied by the original waveform. We treat the sample values (that in reality are now numbers stored in computer memory) as impulses. Mathematically, the spectrum of the sampled signal is now repeated at multiples of the sampling frequency. Taking the Discrete-Time Fourier Transform (DTFT) of the sampled waveform gives the same spectrum as taking the Fourier transform of the impulses representing our samples.

Line 4 The standard way to convert digital values back into an analog signal is to use an R-2R resistor ladder. The bits in a binary value activate or inactivate inputs to the ladder, and the ladder causes the inputs to be binary weighted and superimposed to create an analog output value.

The voltage is held constant until a new sample value arrives at the ladder inputs. The output voltage of the ladder then steps to a new value, meaning the output voltage waveform has a stair-step shape. In the Rosetta Stone diagram, each impulse is turned into a rectangle function by convolving with a single rectangle function. Strictly speaking, the rectangle function we use is non-causal, but this just causes the output waveform to arrive one-half sample earlier, a difference that is inconsequential for the frequency content and waveform shape in the end. By using the non-causal rectangle function the Rosetta Stone diagram is tractable; using a rectangle shifted to $t = 0$ to 1 would create an imaginary component that would clutter up the diagram enormously. Using real and even functions in the Rosetta Stone diagram wherever possible leads to clean results without a loss of critical information.

Line 5 In the Rosetta Stone diagram, convolving the impulse samples with a rectangle function creates the stair-step waveform. If we open up a cellphone, we would find the stair-step waveform being fed into a low-pass filter to smooth it out.

Line 6 The low-pass filter we use is an ideal low-pass filter, represented by a rectangle in the frequency domain. Once again we use a non-causal signal in the time domain for the impulse response of the low-pass filter, thereby achieving a remarkably simply analysis with little loss of fidelity. It also worth noting that, for more accuracy, we may use a Bode plot to find the product of the stair-step signal spectrum and the actual low-pass transfer function.

Line 7 The spectrum of the reconstituted analog signal is the convolution of the stair-step waveform and the low-pass filter impulse response. Equivalently, the reconstituted analog signal is the inverse Fourier transform of the final spectrum on the lower right side.

COMM: The final spectrum is slightly distorted, despite having having eliminated aliasing and using perfect low-pass filters in the analysis. Where is the distortion coming from? The answer lies in Line 4, where we convolved the sample impulses with a rectangle function. The spectrum (a sinc function) of the rectangle causes some suppression of higher frequencies in the baseband copy of the spectrum.

How would we reduce the distortion in the output waveform? The answer is to widen the sinc function in Line 4. A wider sinc function would map to a narrower rectangle in the time domain. That would cause empty space between the stair-steps in the time domain, requiring analog circuitry with a faster response time. In the limit, we would achieve the best results if the rectangle function in Line 4 became an impulse! If we could output impulses from the D/A converter, our music would have higher fidelity. However, we would need much faster circuitry, to say the least.

Instead of using impulses, the practical solution to achieving high fidelity is to use a higher sampling rate. A higher sampling rate would mean we are using narrower stair-steps (but without spaces between them), and our since function in the frequency domain would be come wider and flatter.