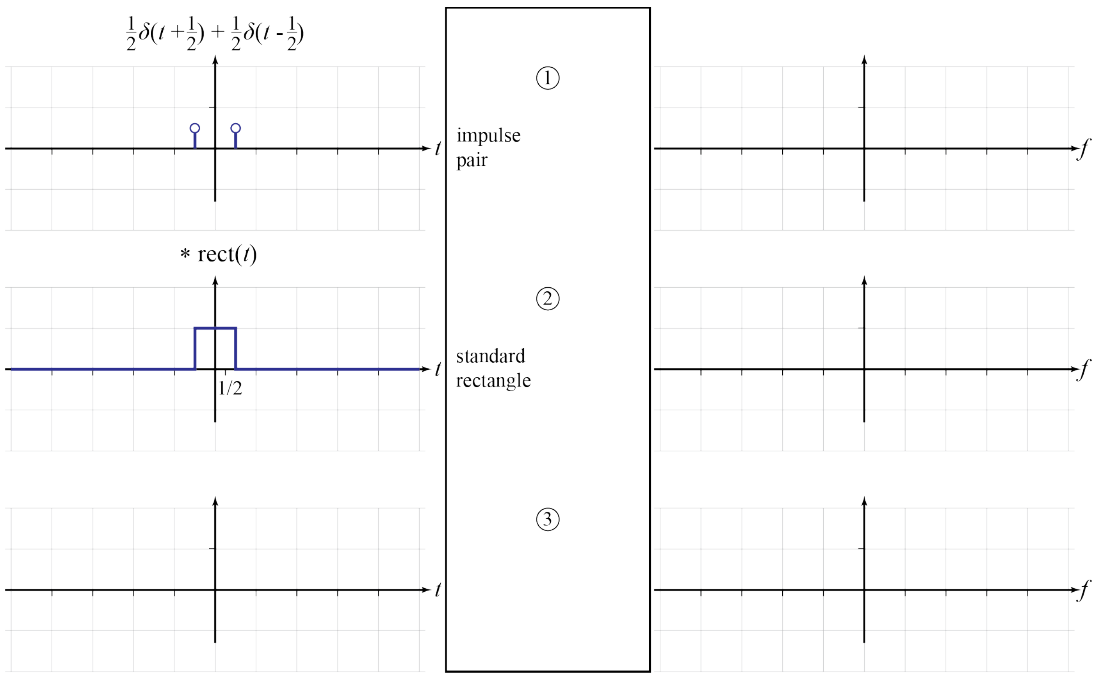
EX: Sketch the missing waveforms and label vertical axes with mathematical expressions to complete both sides of the diagram for the following:

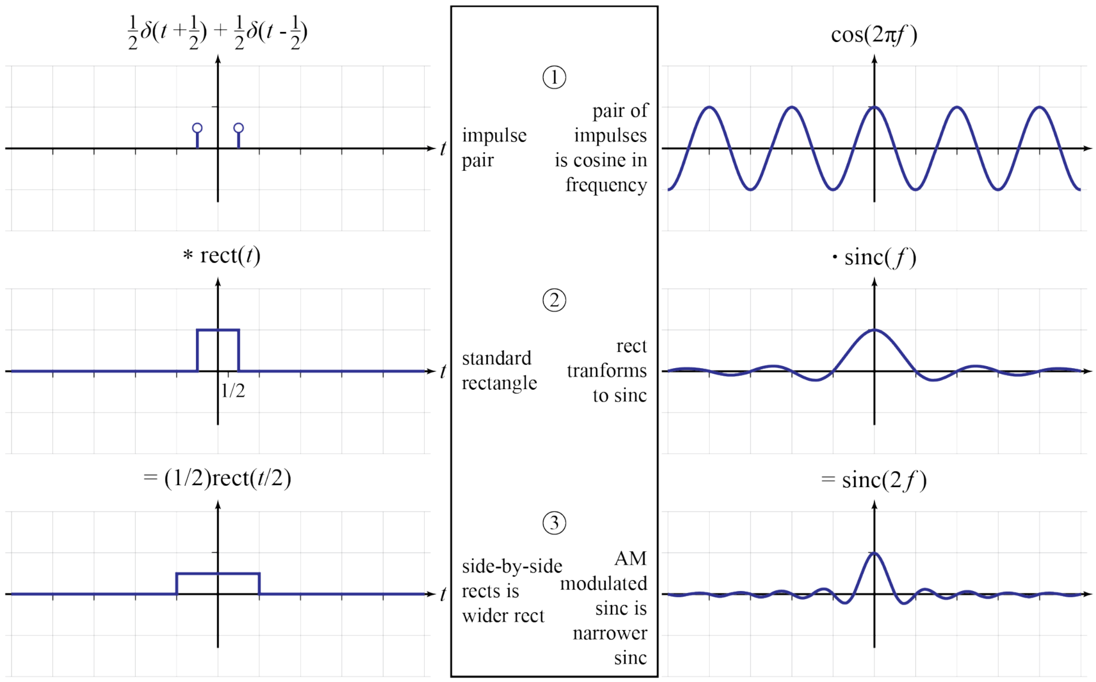
$$\mathscr{F}\left\{\frac{1}{2}\left[\delta\left(t+\frac{1}{2}\right)+\delta\left(t-\frac{1}{2}\right)*\mathrm{rect}(t)\right\}.$$

Hint: To find the third function on the right side, perform the convolution on the left side and then take the Fourier transform.



SOLN: See diagram below. Use Fourier transform pairs and identities to complete the first two lines. The first line is the identity for cosine but in the opposite direction. Since cosine is even and real, the Fourier transform and inverse Fourier transform pair work in both the forward and reverse directions.

For the third line, convolution of an impulse with any function creates a copy of that function with its origin moved to the location of the impulse. Thus, we get two rectangle functions, next to each other. To be precise, based on the definition of the rectangle function as having a value of 1/2 at $t = \pm 1/2$, we have a value of 1 = 1/2 + 1/2 at $t = 0$ for the sum. So we do have a double-wide rectangle as our sum.



On the right side, this example shows that convolution in time becomes multiplication in frequency. As is often the case when using the Rosetta Stone method, rather than using information on just one side of the diagram, we use a Fourier transform or inverse transform to determine a function's shape from the other side of the diagram.

This redundancy of information is a strength of the Rosetta Stone method. Here, we could calculate the waveform on the right side of Line ③ by using the $\text{sinc}(f)$ on Line ② as an envelope for the $\cos(2\pi f)$ on Line ①, which is what we do for Amplitude Modulation (AM). However, transforming the rectangle from the time domain makes it easy to see what the exact function on the right side of Line ③ should be.

REF: Ronald Bracewell, *The Fourier Transform and its Applications*, 2nd Ed., New York, NY: McGraw-Hill, 1978. ISBN 0-07-007013-X