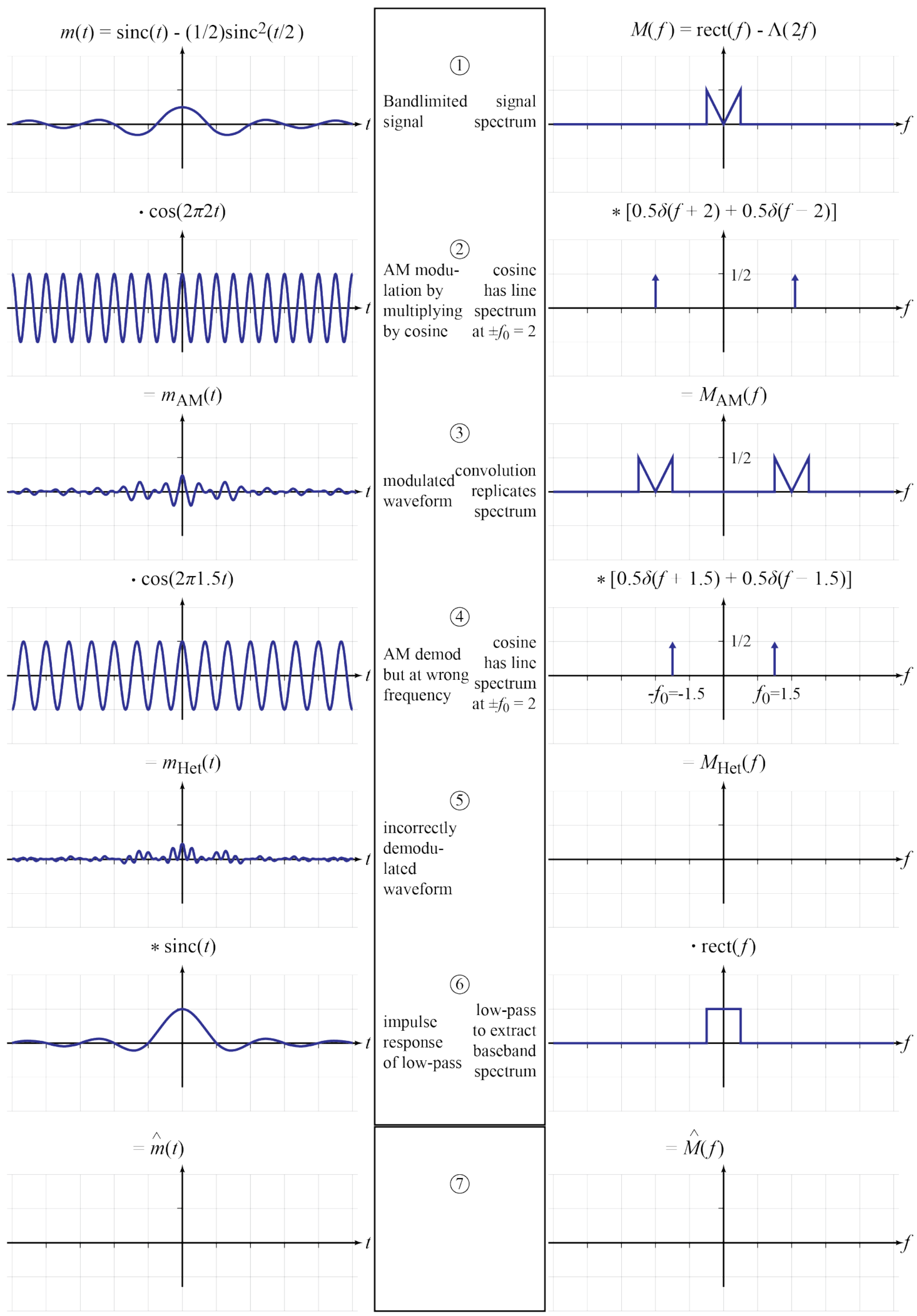
Ex: As shown in the CTool on AM modulation, AM modulation and demodulation may be accomplished by multiplying by the cosine carrier frequency twice and low-pass filtering. This problem explores what happens if the demodulation frequency is erroneous.

An engineer implements a faulty AM modulation system as shown in the diagram below. The fault is that the **second cosine multiplication is a different frequency than the first cosine multiplication**.

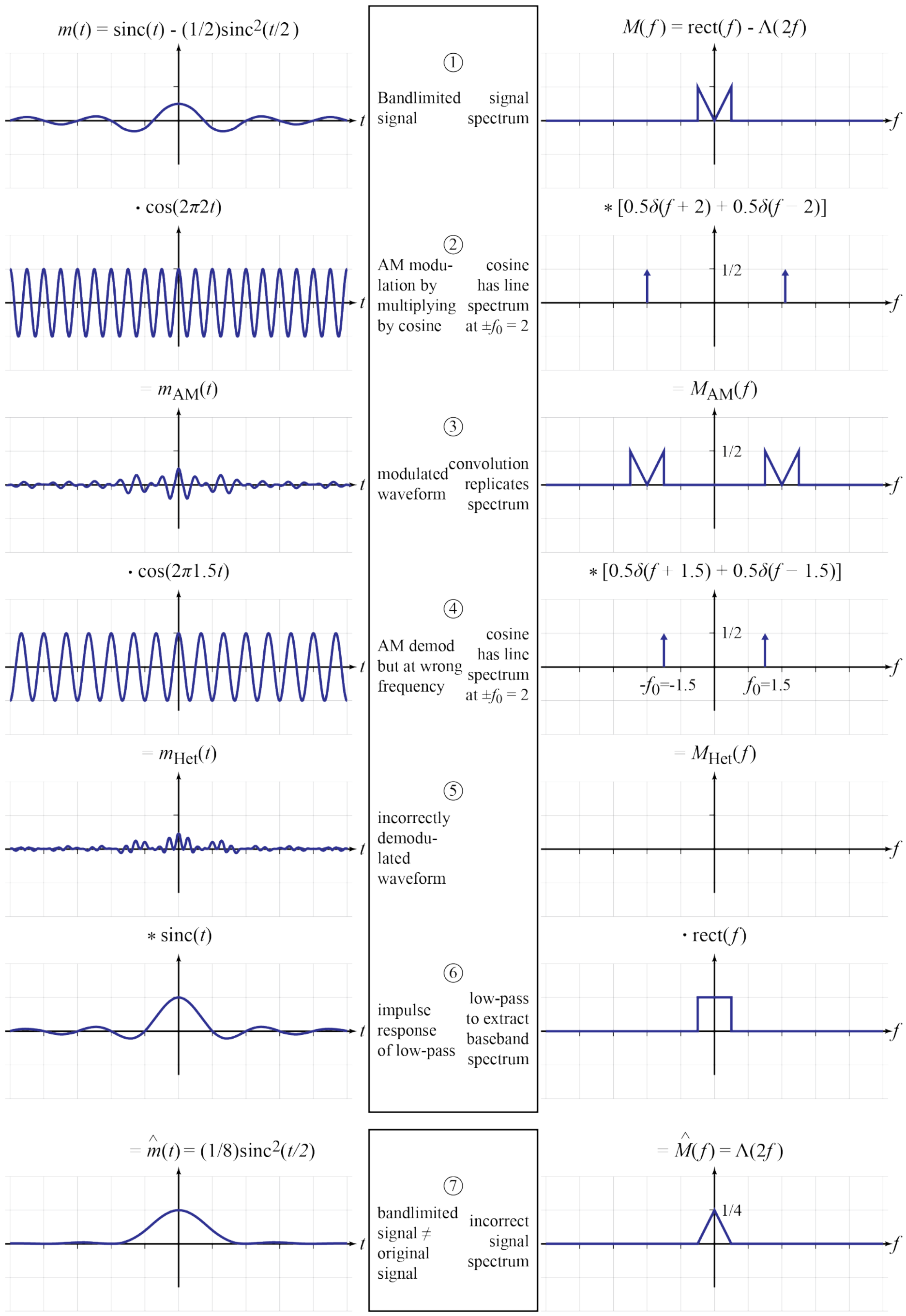
Fill the three missing waveforms on the diagram below (next page) on both the time and frequency sides to determine what is happening in this scheme. You do not have to fill in the center box, and vertical scaling need only be relatively correct (i.e., correct shapes of waveforms). Also, you may omit parts of spectra that lie off the right left ends of the axes provided below.

Hint: The final spectrum on the lower right should be a familiar function that you may inverse transform to get the waveform on the lower left side.

See diagram next page.


m(t) = sinc(t) - (1/2)sinc²(t/2)
M(f) = rect(f) - Λ(2f)
①
Bandlimited signal
signal spectrum
· cos(2π2t)
∗ [0.5δ(f + 2) + 0.5δ(f − 2)]
②
AM modu-lation by multiplying by cosine
cosine has line spectrum at ±f0 = 2
1/2
= mAM(t)
= MAM(f)
③
modulated waveform
convolution replicates spectrum
1/2
· cos(2π1.5t)
∗ [0.5δ(f + 1.5) + 0.5δ(f − 1.5)]
④
AM demod but at wrong frequency
cosine has line spectrum at ±f0 = 2
1/2
-f0=-1.5
f0=1.5
= mHet(t)
= MHet(f)
⑤
incorrectly demodu-lated waveform
∗ sinc(t)
· rect(f)
⑥
impulse response of low-pass
low-pass to extract baseband spectrum
= m̂(t)
= M̂(f)
⑦
t
f

SOLN: To find $M_{Het}(t)$ in Line 5, we just replicate the spectra with their origins translated to $\pm f_0 = \pm 1.5$. In proper AM demodulation, spectral copies at the origin reinforce. Here they end up side-by-side.



For the spectrum in the last line of the diagram, we multiply the functions in Line 5 and Line 6. This operation low-pass filters the signal, and we get a triangle wave, which is nothing like the original spectrum.

The inverse transform of the triangle waveform is a sinc^2 function. This is a standard Fourier transform pair that is symmetrical since the functions are even and real.

At first glance, the final waveform looks similar to the original $m(t)$, but the central peak is higher, and the zero-crossings are different. Part of the spectrum has been eliminated by the erroneous AM demodulation.

REF: Ronald Bracewell, *The Fourier Transform and its Applications*, 2nd Ed., New York, NY: McGraw-Hill, 1978. ISBN 0-07-007013-X