TOOL: The Rosetta Stone method is a unified, graphical way of analyzing Linear, Time-Invariant (LTI) systems, which are ubiquitous in engineering solutions. What the Rosetta Stone method offers is, first, a unified framework that embeds both analog and discrete LTI systems in a continuous Fourier transform framework and, second, provides graphical methods for computing convolutions and Fourier transforms of complete systems.

As a basic example of the Rosetta Stone method, Fig. 1 shows a simple example of what happens to a band-limited signal when it is low-pass filtered.

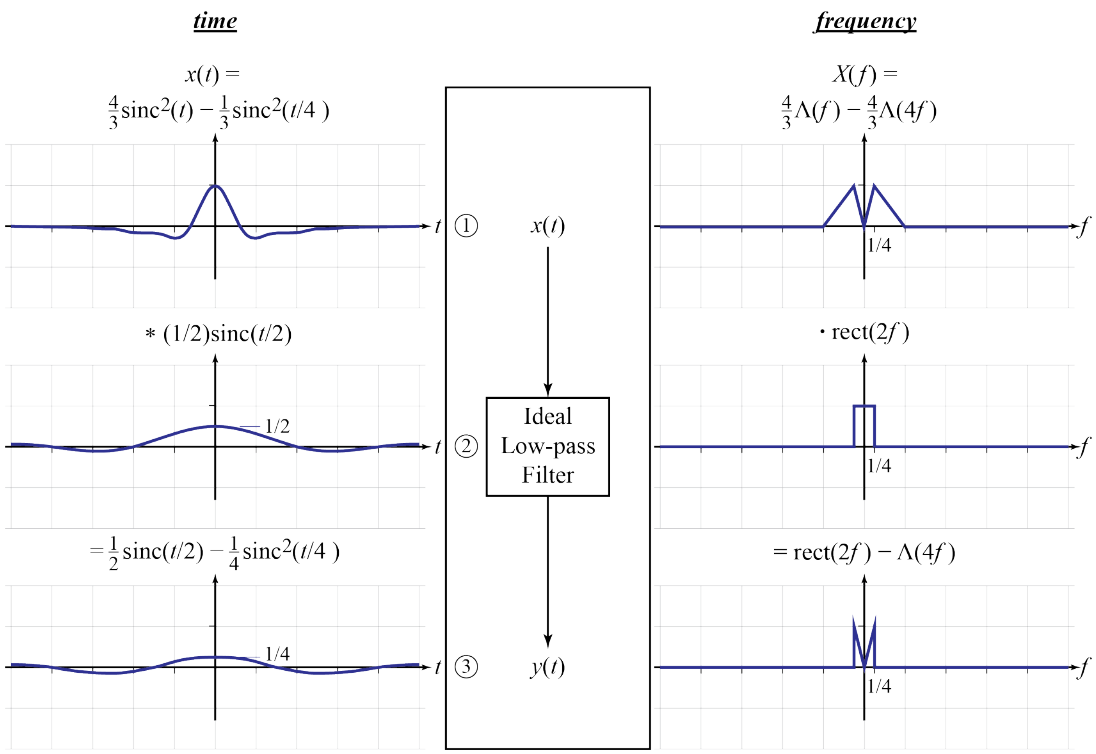

Fig. 1. Rosetta Stone diagram for low-pass filtering of a band-limited signal.

The left side of a Rosetta Stone DSP worksheet shows signals in the time domain. The right side of the Rosetta Stone DSP worksheet shows signals in the frequency domain, i.e., the Fourier transform of the time-domain signals. Line 1 shows a band-limited signal with its Fourier transform on the right. There are several notable features of the signal: 1) the signal is a generic signal created from a convenient Fourier transform pair, namely the sinc^2 function and triangle function; 2) the functions in the Fourier transform pair are real and even, meaning they are a transform pair in both the forward and inverse directions; and 3) the signal in the time domain is non-causal. The latter quality is a departure from the use of causal signals, but the use of non-causal signals delivers much simpler analyses with minimal impact on the relevance of results that focus on how the frequency content of the final signal is shaped by the overall system. Merely assuming a delay is added to the system output is sufficient to mitigate the use of non-causal signals.

What happens in the example of Fig. 1 is that we convolve $x(t)$ with the impulse response, $h(t)$, of the ideal low-pass filter on the left. We therefore multiply $X(f)$ with the transfer function, $H(f)$, on the right. Once again, for the sake of simplicity, we have used a transform pair that is real and even for the ideal low-pass filter. After the multiply on the right, the final spectrum, $Y(f)$, of the system is inverse transformed into a difference of a rectangle function and triangle function. Again, we use a real and even transform pair in the rectangle and sinc functions. This property of the functions used, allows us to write a formula for the time-domain output, $y(t)$. Here, the waveform for that formula was calculated with a graphing program, but we might omit the waveform if we are only interested in the frequency content, $Y(f)$, which is typically the case.

The preceding example illustrates that the utility of the Rosetta Stone method is its ability to give a qualitative, high-level understanding of what an LTI system is doing. As a second example, in a digital music system, the Rosetta Stone method quickly reveals that using a stair-step waveform in the reconstruction stage introduces distortion and that using impulses instead of stair-steps would eliminate the distortion. (See CTools: Rosetta Stone Method: DSP: A/D and D/A). What the Rosetta Stone method accomplishes is providing an analysis with a minimal use of equations and numerical calculations. The analysis involves only sketches that may be made quickly on paper.

A second major component of the Rosetta Stone method is a graphical approach for computing convolutions. As shown in the first example above, convolution in one domain corresponds to multiplication in the other domain. Thus, a simple way of computing convolution is desirable. The Rosetta Stone method of convolution exploits two key ideas: 1) the convolution of an impulse with a function results in a replication of the function, and 2) the convolution of a function with the derivative of another function, followed by integration, gives the same result as direct convolution of the original functions. Fig. 2 shows a basic example.

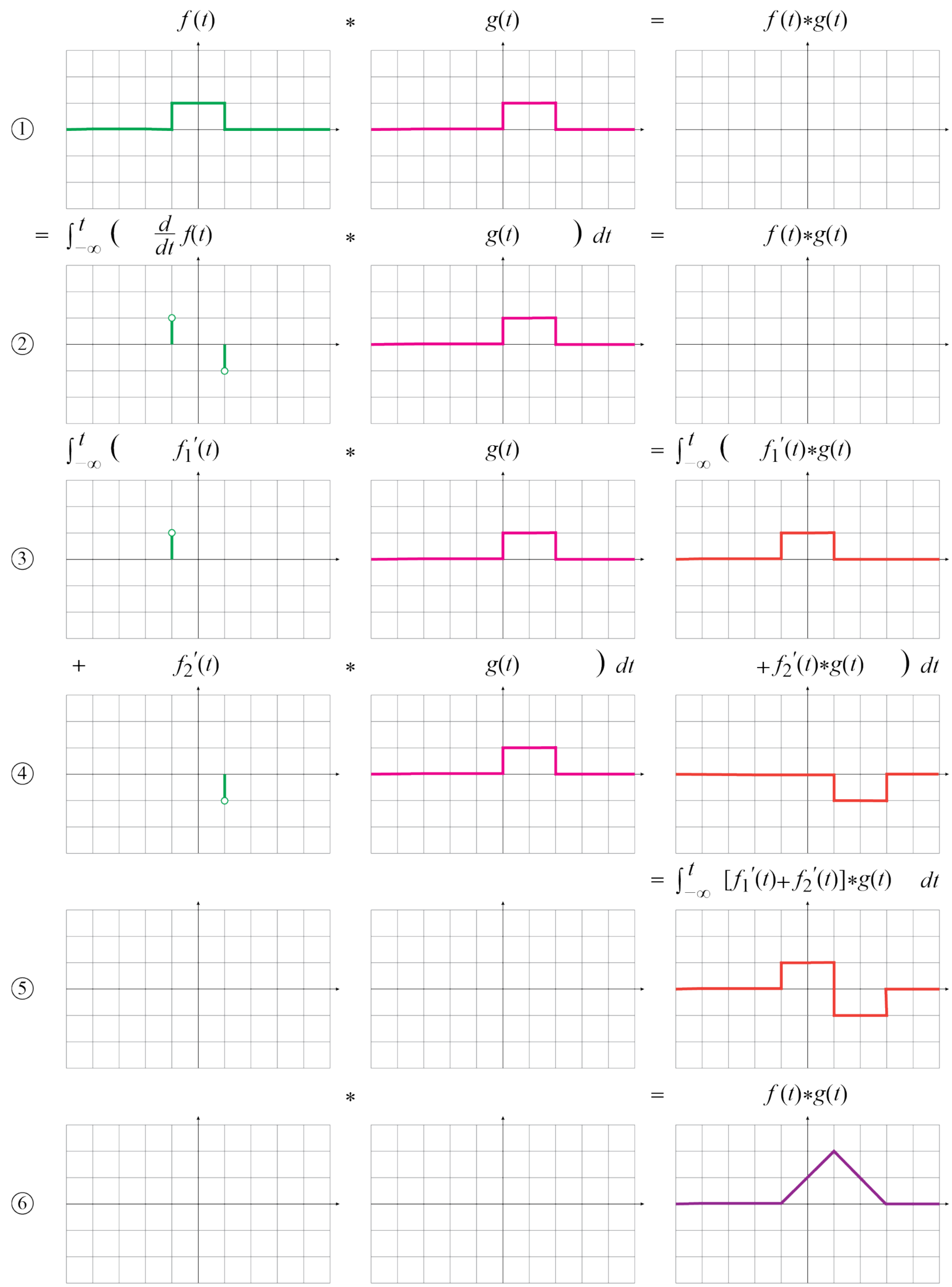

Fig. 2. Example of Rosetta Stone method of convolution.

Line 1 is the convolution we wish to compute, with the answer left blank.

On Line 2, the derivative of $f(t)$ is shown being convolved with $g(t)$ and the result integrated, again with the answer left blank. Note that only one function is differentiated, unlike the product rule for derivatives. Either or both functions may be differentiated one or more times and the result integrated the corresponding number of times.

On Lines 3 and 4, the derivative of $f(t)$ is split into two pieces for clarity. Convolutions are additive, allowing us to decompose the problem into smaller pieces. The right side now shows the two convolved derivative pieces.

Line 5 shows the sum of the convolved derivative pieces, and Line 6 shows the integral of Line 5, which is the final answer.

Note the simplicity of the convolution calculation—no time-reversed and shifted functions or integrals evaluated for every point of time. The Rosetta Stone method reduces the amount of work required to compute the convolution.

Note also that the Rosetta Stone method of convolution may be applied directly to sampled waveforms where we treat sample values as impulses. Again, time-reversals are eliminated. Instead we get a copy of one of the sampled waveforms scaled and shifted according to the sampled values in the other waveform. The Rosetta Stone worksheet facilitates this calculation.

Study Material

The information about the Rosetta Stone method is organized as an outline of brief notes on topics relevant to the method. The reader may access the items in the outline in any order, as needed, but the "introduction" pages are the best place to start learning about the Rosetta Stone method.