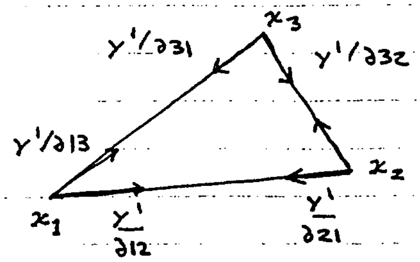
Cubic Splines – N-Dimensions –

Cotter Directional Derivatives for Δ spline

2 Sept 1993



Given $\frac{\partial y}{\partial p_i}$ where p_i is e.g. units per units such as 1 mil / °F.

We first normalize. $p_{max} - p_{min} \Rightarrow 1$ unit

So if we have 1 mil /°F and $°F_{max} - °F_{min} = 500\,°F$

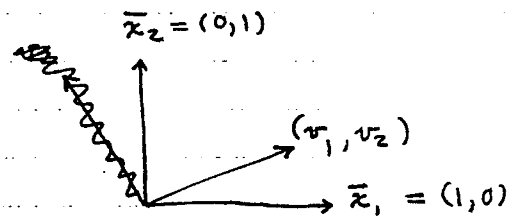
Then 1 mil /°F becomes 500 mils / 500°F.

Thus $\frac{\partial y}{\partial \bar{x}_i} = \frac{\partial y}{\partial p_i}(p_{max} - p_{min})$.

Now we want to transform $\frac{\partial y}{\partial \bar{x}_i}$ derivatives to the directions of line segments connecting points.

ex: $\vec{v}$ in direction from x_1 to x_2 $= \frac{x_2 - x_1}{\sqrt{|x_2 - x_1|^2}}$

$\vec{v}$ is a unit vector $= v_1, v_2$



$v_1^2 + v_2^2 = 1$ $\quad \frac{\partial y}{\partial \vec{v}} = v_1 \frac{\partial y}{\partial \bar{x}_1} + v_2 \frac{\partial y}{\partial \bar{x}_2}$

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In general we have

$$\vec{v} = (v_1, \ldots, v_n) = \frac{x_2 - x_1}{\sqrt{|x_2 - x_1|^2}}$$

$$\frac{\partial y}{\partial \vec{v}} = v_1 \frac{\partial y}{\partial x_1} + \ldots + v_n \frac{\partial y}{\partial x_n}$$

This gives us the derivative in the direction from x_1 to x_2, but we want to normalize so the derivative is in terms of λ's.

$\lambda = 0$ corresponds to $\cancel{\bullet}\, l \cdot \vec{v}$
$\lambda = 1$ " " $0\cancel{l} \cdot \vec{v}$ $\qquad l = \sqrt{|x_2 - x_1|^2}$

Thus $\dfrac{\partial \cancel{\ } y}{\partial \vec{v}} = \dfrac{\partial y(l(1-\lambda)\vec{v})}{\partial \vec{v}}$.

Anyhow $\dfrac{\partial y}{\partial \lambda} = -l \dfrac{\partial y}{\partial \vec{v}}$ since $\Delta\lambda = 1 \leftrightarrow \Delta\vec{v} = -l$.

Let $\bar{u} = x_2 - x_1 = (u_1, \ldots, u_n)$ $\qquad x_1, x_2$ defined pts

$$\frac{\partial y}{\partial \bar{u}} \equiv \frac{\partial y}{\partial 12} = -u_1 \frac{\partial y}{\partial \bar{x}_1} - \ldots - u_n \frac{\partial y}{\partial \bar{x}_n}$$