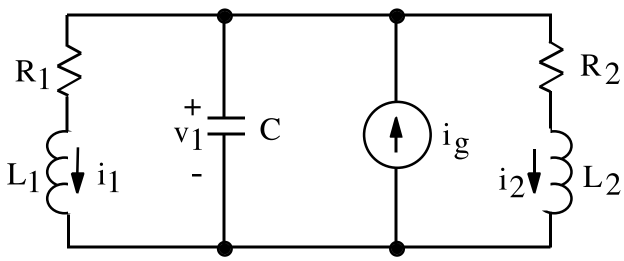
**EX:** Write the first-order coupled differential equations for the circuit below in the form d**x**/d$t$ = f(**x**, $t$) where **x** is the state vector and $t$ is time.




Third-order circuit. $i_g(t)$ switches from $-i_0$ to $i_0$ at $t = 0$.

**ANS:**

$$\frac{di_1}{dt} = \frac{v_1 - i_1 R_1}{L_1}$$

$$\frac{di_2}{dt} = \frac{v_1 - i_2 R_2}{L_2}$$

$$\frac{dv_1}{dt} = \frac{i_o - (i_1 + i_2)}{C}$$

**SOL'N:** The state variables are always the inductor currents and capacitor voltages (which are also the variables we use to calculate stored energy). Thus, our state variables are $i_{L1}$, $i_{L2}$, and $v_{C1}$. We denote these as $i_1$, $i_2$, and $v_1$.

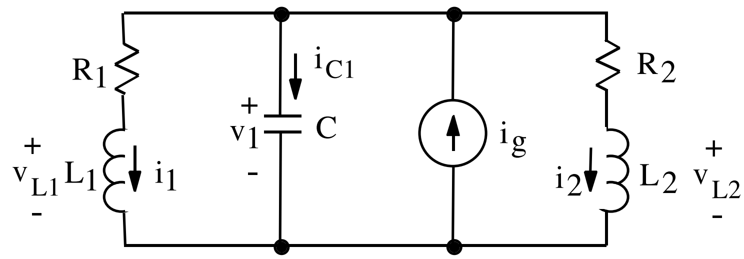
We use the basic component equations to translate derivatives of state variables into non-derivatives:

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

$$\frac{dv_C}{dt} = \frac{i_C}{C}$$

Application of these equations reduces the problem to that of writing equations for $v_{L1}$, $v_{L2}$, and $i_{C1}$. Each of these equations must have only the state variables, $i_{L1}$, $i_{L2}$, and $v_{C1}$, on the other side so the final equations (in terms of the derivatives of state variables) involve only state variables.

The circuit diagram below shows $v_{L1}$, $v_{L2}$, and $i_{C1}$. We now apply Kirchhoff's laws—voltage loops and current sums at nodes—to find our state-space equations.




The equation for $v_{L1}$ must come from a voltage loop, and the voltage loop on the left will suffice in this case. We use Ohm's law to express the voltage across $R_1$:

$$v_{L1} + i_1R_1 - v_1 = 0V \text{ or } v_{L1} = v_1 - i_1R_1$$

(Note that other voltage loops that include $L_1$ would pose difficulties with requiring a voltage for a current source or requiring a second inductor voltage. If the C were missing, however, we would be forced to write an equation including both $v_{L1}$ and $v_{L2}$. In that case, we would also write a current summation equation for the top center node, allowing us to express $i_1$ in terms of $i_2$ and eliminate one state variable. Because we already eliminated one state variable by eliminating C, this would leave us with only one state equation plus an equation relating the two state variables.)

The equation for $v_{L2}$ must also come from a voltage loop, and a voltage loop thru the C will once again suffice. We use Ohm's law to express the voltage across $R_2$:

$$v_1 - v_{L2} - i_2R_2 = 0V \text{ or } v_{L2} = v_1 - i_2R_2$$

The equation for $i_{C1}$ must come from a current summation. In this circuit, there are only two nodes: the top and the bottom rails. Because they effectively yield the same current summation equation, we only use one of these nodes. For the top node, summing currents flowing out of the node gives the following equation:

$$i_1 + i_{C1} - i_g + i_2 = 0A \text{ or } i_{C1} = -i_1 + i_g - i_2$$

To complete the derivation, we use the basic component equations to change $v_{L1}$, $v_{L2}$, and $i_{C1}$ back into derivatives of state variables.

$$\frac{di_1}{dt} = \frac{v_1 - i_1R_1}{L_1}$$

$$\frac{di_2}{dt} = \frac{v_1 - i_2 R_2}{L_2}$$

$$\frac{dv_1}{dt} = \frac{-i_1 + i_o - i_2}{C}$$

Note that in the third equation we have substituted the value of $i_g$ for $t > 0$.