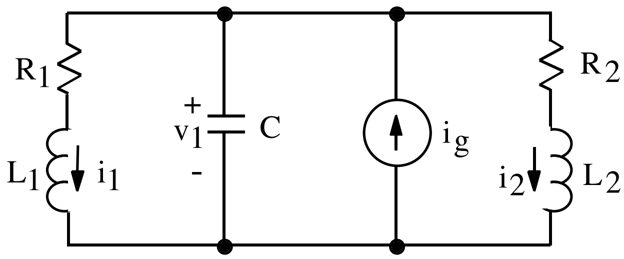
EX: Evaluate the state vector **x** at t = 0+ for the circuit below in terms of symbolic component names.



Third-order circuit. $i_g(t)$ switches from $-i_0$ to i_0 at $t = 0$.

ANS:

$$i_1(0^+) = \frac{-i_0 R_2}{R_1 + R_2}$$

$$i_2(0^+) = \frac{-i_0 R_1}{R_1 + R_2}$$

$$v_1(0^+) = -i_0 \cdot R_1 \parallel R_2$$

SOL'N: The state variables are always the inductor currents and capacitor voltages (which are also the variables we use to calculate stored energy). Thus, our state variables are i_{L1}, i_{L2}, and v_{C1}. We denote these as i_1, i_2, and v_1.

Because their values cannot change instantly, the state variables have the same values at time $t = 0^-$ as they do at time $t = 0^+$.

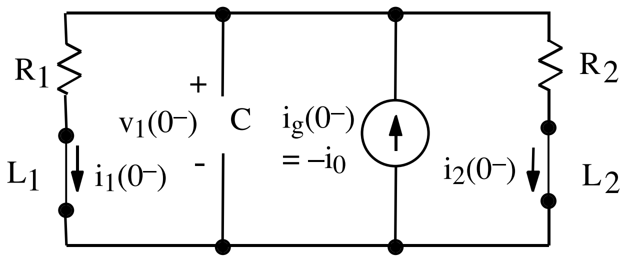
Because the circuit has had the same DC current source for an infinitely long time at $t = 0^-$, (from time $-\infty$ to 0^-), the circuit will have reached equilibrium and time derivatives of state variables will be zero. In other words, currents and voltages are no longer changing.

Thus, we have that v_{L1}, v_{L2}, and i_{C1} are all zero based on the basic component equations:

$$\frac{di_L}{dt} = \frac{v_L}{L}$$

$$\frac{dv_C}{dt} = \frac{i_C}{C}$$

This means the inductors look like wires and the capacitors look like open circuits. We get the equivalent circuit shown below at time $t = 0^-$.



The circuit is now a simple current divider, and the capacitor voltage is given by Ohm's law (using i_g and $R_1 || R_2$).

$$i_1(0^-) = -i_0 \frac{R_2}{R_1 + R_2}$$

$$i_2(0^-) = -i_0 \frac{R_1}{R_1 + R_2}$$

$$v_1(0^-) = -i_0 \cdot R_1 \parallel R_2$$

These are the same values as the initial conditions at $t = 0^+$.