EX: Three different systems for orienting a solar panel toward the sun have been tested. Four samples of the power output of the solar panel were taken using each system. The results are as follows, (all values in kW):

$x_{11} = 50 \quad x_{12} = 52 \quad x_{13} = 48 \quad x_{14} = 52 \quad x_{15} = 48$

$x_{21} = 47 \quad x_{22} = 47 \quad x_{23} = 51 \quad x_{24} = 55 \quad x_{25} = 55$

$x_{31} = 58 \quad x_{32} = 40 \quad x_{33} = 40 \quad x_{34} = 58 \quad x_{35} = 49$

Use ANOVA to determine whether to accept the following null hypothesis, H_0, or the alternate hypothesis, H_1, at the 1% significance level.

H_0: $\mu_1 = \mu_2 = \mu_3$

H_1: At least one of the means is not equal to the others

SOL'N: We have $k = 3$ different treatments since we have three different systems.

For each system, we have $n_i = n = 5$ observations.

This gives us $N = k \cdot 5 = 15$ observations in all.

Now we calculate the means of the observations for each system:

$$\bar{x}_{1\bullet} = \frac{1}{n_1}\sum_{j=1}^{n_i} x_{1j} = \frac{50+52+48+52+48}{5} = 50$$

$$\bar{x}_{2\bullet} = \frac{1}{n_2}\sum_{j=1}^{n_i} x_{2j} = \frac{47+47+51+55+55}{5} = 51$$

$$\bar{x}_{3\bullet} = \frac{1}{n_3}\sum_{j=1}^{n_i} x_{3j} = \frac{58+40+40+58+49}{5} = 49$$

The calculated mean for all observations from all samples may be calculated as a weighted average of sample means or as the average of all observations:

$$\bar{x}_{\bullet\bullet} = \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{n_i} x_{ij} = \frac{1}{N}\sum_{i=1}^{k} n_i \bar{x}_i = \frac{1}{15}(5\cdot 50 + 5\cdot 51 + 5\cdot 49) = 50$$

Next, we calculate the squares of the variations around the sample means:

$$SSA = \sum_{i=1}^{k} n_i(\bar{x}_{i.} - \bar{x}_{..})^2 = 5 \cdot 0 + 5 \cdot 1 + 5 \cdot 1 = 10$$

$$SSE = \sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_{i.})^2 = \sum_{j=1}^{n_1}(x_{1j} - \bar{x}_{1.})^2 + \sum_{j=1}^{n_2}(x_{2j} - \bar{x}_{2.})^2 + \sum_{j=1}^{n_3}(x_{3j} - \bar{x}_{3.})^2$$

$$= (0+4+4+4+4) + (16+16+0+16+16) + (81+81+81+81+0)$$

$$= 16 + 64 + 324 = 404$$

The calculated mean of the sum of all treatment squared errors is found by dividing SSA by the appropriate degrees of freedom:

$$MSA = \frac{SSA}{k-1} = \frac{10}{2} = 5$$

Likewise, the calculated mean of the sum of squared errors is found by dividing SSA by the appropriate degrees of freedom:

$$MSE = \frac{SSE}{N-k} = \frac{404}{12} \cong 33.67$$

The ratio of MSA and MSE gives the f value for testing the hypothesis:

$$f = \frac{MSA}{MSE} \sim F_{k-1,N-k} = F_{2,12} \text{ distribution}$$

$$f = \frac{5}{33.67} = 0.1485$$

Using Table A.6 in [1] with $\alpha = 0.01$, we have the following critical value for f:

$$f_{\alpha=0.01}(\nu_1\text{=}2,\ \nu_2\text{=}12) = 6.93$$

For us to reject the null hypothesis, we would have to have $f > 6.93$. This is *not* the case, as $f = 0.1485$. Thus, we accept the null hypothesis and assume the means are all the same.

REF: [1] Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability and Statistics for Engineers and Scientists,* 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.