EX: An engineer is analyzing a circuit model for a biological system in which many resistances representing tissue types will be in series. The engineer measures values of resistance for many tissue samples and finds that the average is $\mu = 20$ kΩ with a variance of $\sigma^2 = (1 \text{ k}\Omega)^2$. For reasons relating to power absorption by tissue, the engineer wishes to find the probability, $P_{990\text{k}\Omega}$, that 50 tissue elements in series will have a total resistance that is less than 990 kΩ. Estimate $P_{990\text{k}\Omega}$.

PROOF: By the central limit theorem, the distribution of the sum, X, of the 50 resistances in series is approximately a gaussian distribution:

$$X{\sim}N(50\mu, 50\sigma^2)$$

NOTE: This involves an assumption that the resistance measurements represent independent random variables. Because problems become intractable otherwise, we often invoke such an assumption without justification.

NOTE: When we use the central limit theorem we typically have no idea how good our approximation is. The rule of thumb, however, is that the sum of 30 or more independent samples from the same distribution will be sufficiently close to a gaussian that any residual error may be politely ignored.

We convert the value of interest, 990 kΩ, to the equivalent number of standard deviations from the mean for a standard gaussian random variable, Z:

$$Z = \frac{990 \text{ k}\Omega - \mu_X}{\sigma_X} = \frac{990 \text{ k}\Omega - 50 \cdot 20 \text{ k}\Omega}{\sqrt{50 \cdot (1 \text{ k}\Omega)^2}} = \frac{-10 \text{ k}\Omega}{\sqrt{50} \cdot 1 \text{ k}\Omega} = \frac{-10}{\sqrt{50}}$$

or

$$Z \approx -1.414$$

Using a table of cumulative probability, $F(z)$, for a standard gaussian, we find the value of $P_{990\text{k}\Omega}$ as the area in the gaussian tail left of -1.414:

$$P_{990\text{k}\Omega} = F(z = -1.414) = 0.0787$$