EX: An engineer measures the following beta values for bipolar transistors with the aim of finding nominal values of gain, (i.e., beta), to list on a datasheet.

$\beta_1 = 111$ $\beta_2 = 136$ $\beta_3 = 159$ $\beta_4 = 141$ $\beta_5 = 109$ $\beta_6 = 121$

$\beta_7 = 117$ $\beta_8 = 105$ $\beta_9 = 99$ $\beta_{10} = 102$

The variance is known to be $\sigma = 20$. Using the above data, find the confidence interval for the true mean at the 1 % significance level (a.k.a. the 99 % confidence interval). (Assume the underlying distribution for the measured data is gaussian.)

SOL'N: The confidence interval is a function of the sample mean, $\bar{x}$, the level of significance, $\alpha = (100\%-99\%)/100\% = 0.01$, the standard deviation, σ, the number of data points, $n = 10$, and the critical point of the standard normal distribution, $z_{\frac{\alpha}{2}}$.

$$\left(\bar{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \ \bar{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

The value of is found with a spreadsheet to be 120.

For the critical point, we use a standard normal cumulative distribution table:

$$z_{\frac{\alpha}{2}} = z_{0.005} = 2.58$$

Using these values we find the confidence interval:

$$\left(120 - 2.58 \cdot \frac{20}{\sqrt{10}}, \ 120 + 2.58 \cdot \frac{20}{\sqrt{10}}\right) \approx (120 - 16.317, \ 120 + 16.317)$$

or

$$(103.683, \ 136.317)$$

REF: Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability and Statistics for Engineers and Scientists,* 7th Ed., Upper Saddle River, NJ: Prentice Hall, 2002.