TOOL: The signed rank test is a nonparametric test of the null hypothesis that the unknown mean, μ, of a process is equal to the known mean, μ_0, of another process:

H_0: $\mu = \mu_0$

H_1: $\mu \neq \mu_0$

To use the signed rank test, we require a symmetric probability density function, (i.e., distribution), for observations so the median and the mean are equal.

First, we sample the new process N times, obtaining observations x_i. From the x_i and the known mean μ_0 of the second process, we compute the distances, d_i, of the observations from μ_0:

$$d_i = x_i - \mu_0$$

Second, we discard samples equal to μ_0, if any, and reduce N accordingly.

Third, we rank the distances, d_i, in order of their absolute value (i.e., magnitude) from lowest, (rank = 1), to highest, (rank = N). If two or more d_i have the same magnitude, they are all assigned the average value of their ranks. If four d_i's have the same magnitude and correspond to ranks 3, 4, 5, and 6, for example, then all four d_i's are assigned a rank of $(3 + 4 + 5 + 6)/4 = 4.5$.

Fourth, we compute the sum, w_-, of the ranks for negative d_i, and the sum, w_+, of the ranks for positive d_i.

$w_- \equiv$ sum of ranks of $d_i < 0$

$w_+ \equiv$ sum of ranks of $d_i > 0$

Fifth, we define w to be the smaller of w_- and w_+:

$w \equiv$ lesser of w_- and w_+

Sixth, we compare w with a critical value, w_α, from a table, such as Table A.17 of [1], for the desired significance level, α. If $w \leq w_\alpha$, then we reject H_0.

NOTE: We can also test one-sided alternative hypotheses as follows:

H_1: $\mu < \mu_0$ reject H_0 if $w_+ \leq w_\alpha$

H_1: $\mu > \mu_0$ reject H_0 if $w_- \leq w_\alpha$

DEFINITIONS:

μ_0 ≡ known mean of existing process

μ ≡ unknown mean of new process

N ≡ total number of observations (data points) made

i ≡ index designating which sample (treatment) is being considered

x_i ≡ value measured for observation (data point) i

d_i ≡ distance of observation (data point) i from known mean, μ_0

w_- ≡ sum of ranks of observations with negative distances

w_+ ≡ sum of ranks of observations with positive distances

w ≡ lesser of w_- and w_+

w_α ≡ critical value of w for rejecting H_0

α ≡ significance level for rejecting null hypothesis

DERIV: Following the description presented in [2], the signs of the d_i may be thought of as outcomes of Bernoulli trials. That is, the signs are either positive or negative. If $\mu = \mu_0$, then positive and negative signs are equally likely. Thus, $p = q = 1/2$ for the attendant Binomial distribution. (Note that this requires a symmetric distribution so that the mean and median have the same value.)

Taking the logic a step further, the x_i are equally likely to have any rank. This means that the probability of any one particular pattern of + and – signs for the ranked distances is $1/2^N$.

If we start with the most extreme patterns in terms of w_- or w_+ values, such as all –'s or all +'s and work toward the least extreme pattern of half –'s and half +'s, we can determine what value of w_- or w_+ encompasses a set of patterns with total probability as close as possible to α without exceeding it. This yields the critical value, w_α.

NOTE: Finding w_α is straightforward but tedious. Thus, it is convenient to look up w_α values in a table.

REF: [1] Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability and Statistics for Engineers and Scientists,* 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.

[2] Anthony J. Hayter, *Probability and Statistics for Engineers and Scientists,* 2th Ed., Pacific Grove, CA: Duxbury, 2002.