

**Ex:** An electrical engineer is designing integrated circuits for a new type of cell phone that incorporates functions of a personal digital assistant, an MP3 player, and a web browser. The line widths of transistors on the chips are so small that leakage currents have a significant impact on battery life. The engineer is trying to determine whether chips from the center of a wafer have less average leakage compared to chips on the outer edge of the wafer. The leakage for chips in the center of the wafer, where process parameters are tightly controlled, is known to have a mean value of  $\mu_0 = 12$  nA/transistor.

Given that leakage values are distributed symmetrically around the mean value, use the signed rank test to determine whether the following null hypothesis, that the unknown mean leakage,  $\mu$ , of chips on the edge of the wafer is equal to the known mean,  $\mu_0 = 12$  nA/transistor, of chips in the center of the wafer should be rejected at a significance level of  $\alpha = 0.05$ :

$$H_0: \mu = \mu_0$$

$$H_1: \mu > \mu_0$$

The leakage is measured for 25 chips on the edge of the wafer, with the following results, (in nA/transistor):

$$\begin{array}{cccccc} x_1 = 10 & x_2 = 18 & x_3 = 11 & \boxed{x_4 = 12} & x_5 = 15 & \\ x_6 = 16 & x_7 = 14 & x_8 = 13 & x_9 = 15 & x_{10} = 16 & \\ \boxed{x_{11} = 12} & x_{12} = 20 & x_{13} = 10 & x_{14} = 17 & x_{15} = 9 & \\ x_{16} = 17 & x_{17} = 14 & \boxed{x_{18} = 12} & x_{19} = 14 & x_{20} = 15 & \\ x_{21} = 18 & \boxed{x_{22} = 12} & x_{23} = 19 & x_{24} = 10 & \boxed{x_{25} = 12} & \end{array}$$

We discard the values  $\mu_0$  (in boxes) and compute the signed distances,  $d_i = x_i - \mu_0$ :

$$\begin{array}{cccccc} d_1 = -2 & d_2 = +6 & d_3 = -1 & & d_5 = +3 & \\ d_6 = +4 & d_7 = +2 & d_8 = +1 & d_9 = +3 & d_{10} = +4 & \\ & d_{12} = +8 & d_{13} = -2 & d_{14} = +5 & d_{15} = -3 & \\ d_{16} = +5 & d_{17} = +2 & & d_{19} = +2 & d_{20} = +3 & \\ d_{21} = +6 & & d_{23} = +7 & d_{24} = -2 & & \end{array}$$

We list the distances from smallest to largest magnitudes, and we label their ranks. Equal magnitudes receive the average value of corresponding ranks.

-1 (1.5)	+1 (1.5)	-2 (5.5)	-2 (5.5)	-2 (5.5)
+2 (5.5)	+2 (5.5)	+2 (5.5)	-3 (10.5)	+3 (10.5)
+3 (10.5)	+3 (10.5)	+4 (13.5)	+4 (13.5)	+5 (15.5)
+5 (15.5)	+6 (17.5)	+6 (17.5)	+7 (19)	+8 (20)

We compute the sum,  $w_-$ , of the ranks for negative  $d_i$ .

$$w_- = 1.5 + 5.5 + 5.5 + 5.5 + 10.5 = 28.5$$

We reject  $H_0$  if  $w_- \leq w_{\alpha=0.05}$  where  $w_{0.05}$  is the critical value from Table A.17 of [1] for  $N = 20$  (since five points equal to  $\mu_0$  were discarded) and a one-sided test:

$$w_{0.05} = 60$$

Since  $w_- = 28.5 < w_{\alpha=0.05} = 60$ , we reject  $H_0$  and conclude that chips from the edge of a wafer have more leakage than chips from the center of the wafer.

**REF:** [1] Ronald E. Walpole, Raymond H. Myers, Sharon L. Myers, and Keying Ye, *Probability and Statistics for Engineers and Scientists*, 8th Ed., Upper Saddle River, NJ: Prentice Hall, 2007.