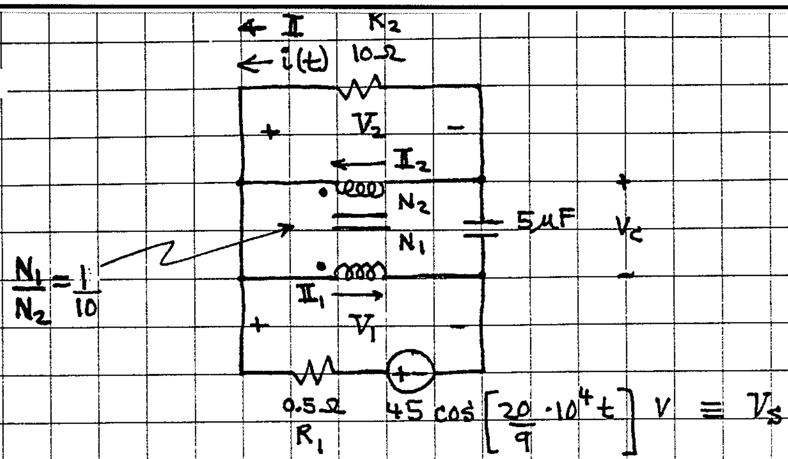


Given an ideal transformer in the above circuit, calculate $i(t)$.

ans: $i(t) = 6\cos\left(\frac{20}{9}\cdot 10^4 t + 143^\circ\right)$ A

soln: As shown in the Text, we have the following relationships for currents and voltages:

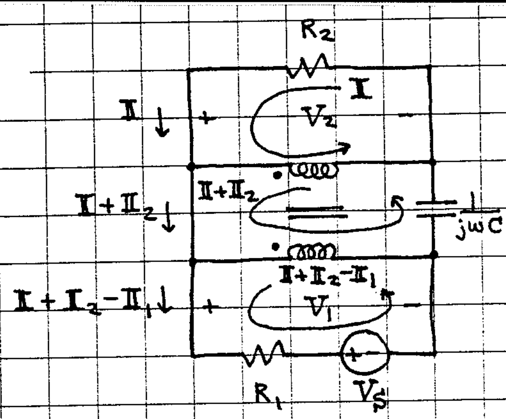
$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\text{or } V_2 = \frac{N_2}{N_1} V_1 \qquad \text{or } I_2 = \frac{N_1}{N_2} I_1$$

Note: We have positive signs in these eq'ns if we define V_1, V_2, I_1, and I_2 as shown in the diagram above. In particular I_1 is in the direction of the passive sign convention, but I_2 is opposite the passive sign convention. Also, V_1 and V_2 are measured with the plus sign on the dotted side of transformer windings.

For the ideal transformer, we do not know what the impedances inside the transformer are. Thus, we must use only the ideal transformer relationships to write eq'ns from which we solve for I (or $i(t)$).

We write mesh current eq'ns for the three loops and then we the ideal transformer relationships.



- We have current $\mathbb{I}$ on the outside of the top loop.
- We have current $\mathbb{I}+\mathbb{I}_2$ on the outside left side of the middle loop.
- We have current $\mathbb{I}+\mathbb{I}_2-\mathbb{I}_1$ on the outside of the bottom loop

Mesh loop eq'ns from top to bottom:

$$-\mathbb{I}\cdot R_2 - V_2 = 0 \quad \text{or} \quad \mathbb{I}R_2 + V_2 = 0 \qquad (1)$$

$$V_2 - V_1 - \frac{1}{j\omega C}(\mathbb{I}+\mathbb{I}_2) = 0 \qquad (2)$$

$$V_1 - R_1(\mathbb{I}+\mathbb{I}_2-\mathbb{I}_1) - V_s = 0 \qquad (3)$$

Use ideal transformer relationships to eliminate $\mathbb{I}_2$ and V_2: $\mathbb{I}_2 = \frac{N_1}{N_2}\mathbb{I}_1 = \frac{\mathbb{I}_1}{10}$

Also, $\frac{1}{j\omega C} = -j9$ $\qquad V_2 = \frac{N_2}{N_1}V_1 = 10V_1$

$$\mathbb{I}R_2 + 10V_1 = 0 \quad \text{or} \quad V_1 = -\frac{R_2}{10}\mathbb{I} = -\mathbb{I} \qquad (1)$$

$$10V_1 - V_1 + j9\left(\mathbb{I} + \frac{\mathbb{I}_1}{10}\right) = 0 \qquad (2)$$

$$V_1 - R_1\left(\mathbb{I} + \frac{\mathbb{I}_1}{10} - \mathbb{I}_1\right) - V_s = 0 \qquad (3)$$

Now use (1) to eliminate V_1 in (2) and (3):

$$\left(-10 + 1\right)\mathbb{I} + j9\mathbb{I} + j\frac{9}{10}\mathbb{I}_1 = 0 \qquad (2)$$

$$-\mathbb{I} - R_1\left(\mathbb{I} - \frac{9}{10}\mathbb{I}_1\right) - V_s = 0 \qquad (3)$$

Now use (2) to express $\mathbb{I}_1$ in terms of $\mathbb{I}$:

$$\mathbb{I}_1 = j\frac{10}{9}(-9+j9)\mathbb{I} = j10(-1+j)\mathbb{I}$$

Substitute for $\mathbb{I}_1$ in (3):

$$-\mathbb{I} - \frac{1}{2}\left(\mathbb{I} - \frac{9}{\cancel{10}} j\cancel{10}(-1+j)\mathbb{I}\right) - V_s = 0$$

$$\left[-\frac{3}{2} + \frac{9}{2} j(-1+j)\right]\mathbb{I} - V_s = 0$$

$$\left(-\frac{3}{2} - \frac{9}{2} - j\frac{9}{2}\right)\mathbb{I} - V_s = 0$$

$$\left(-6 - j\frac{9}{2}\right)\mathbb{I} - V_s = 0$$

$$\mathbb{I} = \frac{V_s}{-6 - j\frac{9}{2}} = \frac{-45}{6 + j\frac{9}{2}} = \frac{-15}{2 + j\frac{3}{2}}$$

$$" = \frac{-30}{4+j3} = \frac{-30}{25}(4-j3) = -\frac{6}{5}(4-j3)$$

$$" = -\frac{6}{5} \cdot 5\angle -37^\circ$$

$$" = \frac{6}{\cancel{5}} \cdot \cancel{5}\angle 180^\circ - 37^\circ$$

$$\mathbb{I} = 6\angle 143^\circ \text{ A}$$

$$\therefore i(t) = 6\cos(\omega t + 143^\circ) \text{ A} \quad \omega = \frac{20}{9}\cdot 10^4 \text{ rad/s}$$