**TOOL:** The transformations below yield a model that absorbs the Early resistance of a BJT, in the DC case, into a model similar to one without Early resistance.

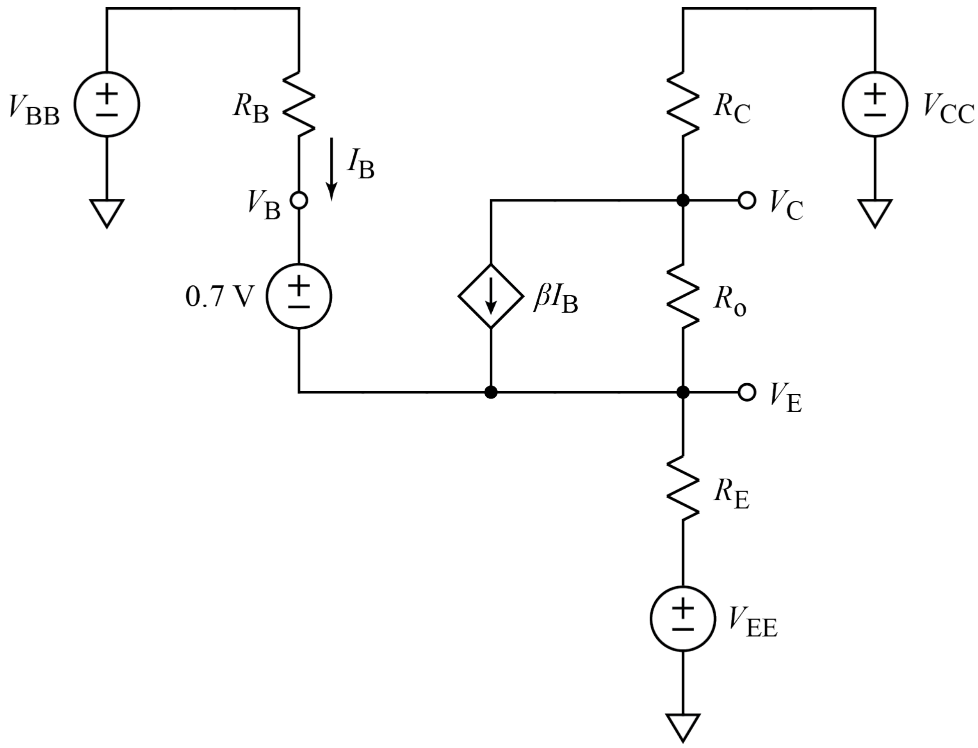


Fig. 1. Generic model for one-transistor circuit.

Early resistance is given by formula in terms of Early voltage from datasheet for transistor and voltage from collector to emitter.

$$R_o = \frac{V_A + V_{CE}}{I_C}$$

Since $V_{CE}$ depends on $R_o$, the equations for the DC model must be used iteratively to find the true DC conditions of the circuit. The first iteration is presented here.

The solution to finding DC conditions involves transforming the dependent source and Early resistance back and forth from Norton form to Thévenin form, collecting more resistors into the portion being transformed. Fig. 2 shows the first step.

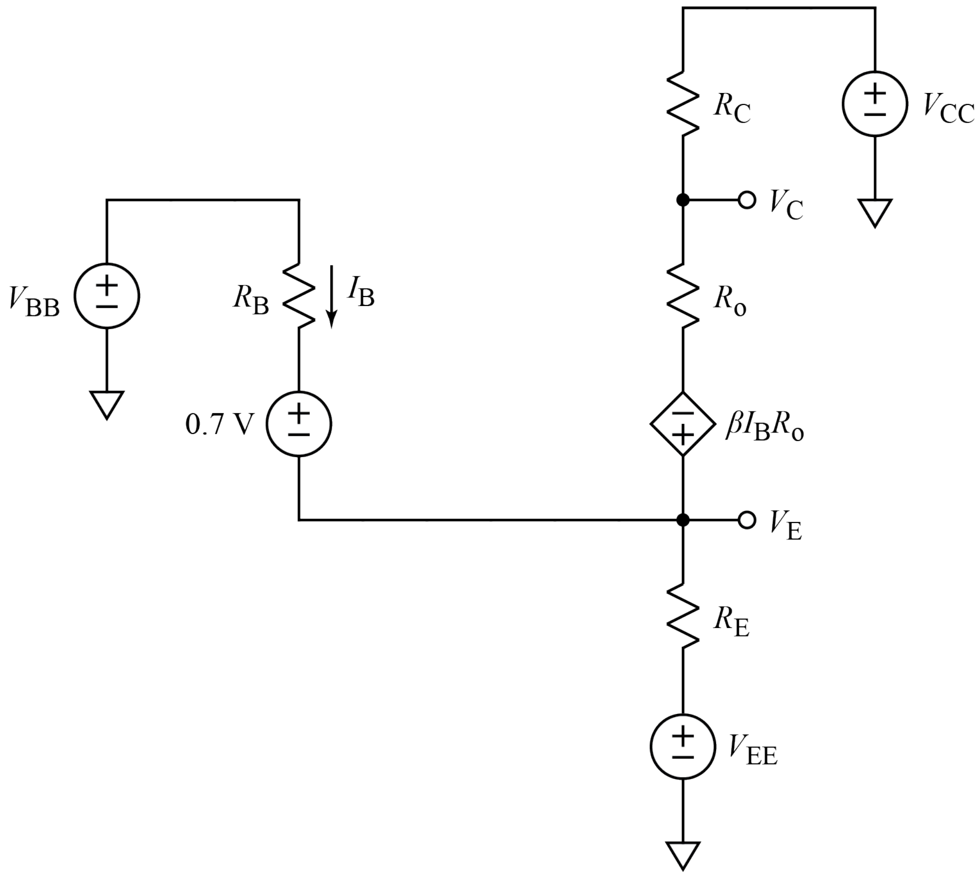


Fig. 2. Dependent source transformed to Thévenin equivalent.

We may transform the dependent source into a Thévenin form here because the transformation leaves current $i_B$ unchanged. That is, we have safely avoided a transformation that would alter the value that the dependent source depends on.

Next, we observe that we may consider $R_o + R_C$ to be the Thévenin resistance that goes with the dependent voltage source, and we may convert this expanded Thévenin equivalent back to a Norton form. Fig. 3 shows the result.

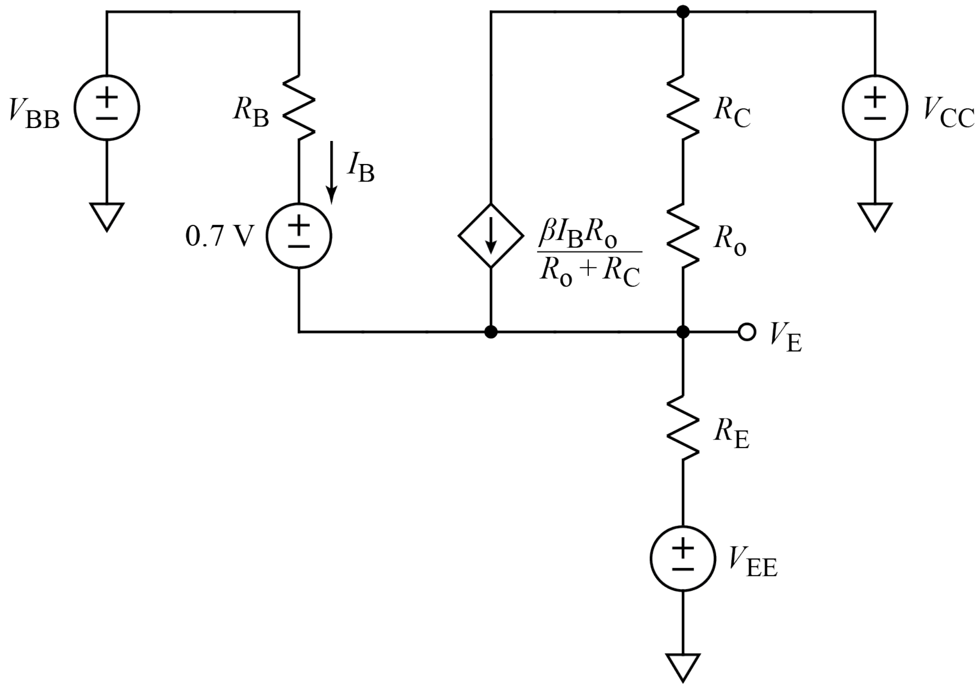


Fig. 3. Thévenin equivalent form, including $R_C$, transformed to Norton equivalent form.

Since both the dependent source and $R_C$ go to independent voltage source $V_{CC}$, we may assume both the dependent source and $R_C$ have their own $V_{CC}$ source. We also define a modified beta value in Fig. 4 for the transistor, based on the multiplier in the dependent source.

$$\beta' = \frac{\beta R_o}{R_o + R_C}$$

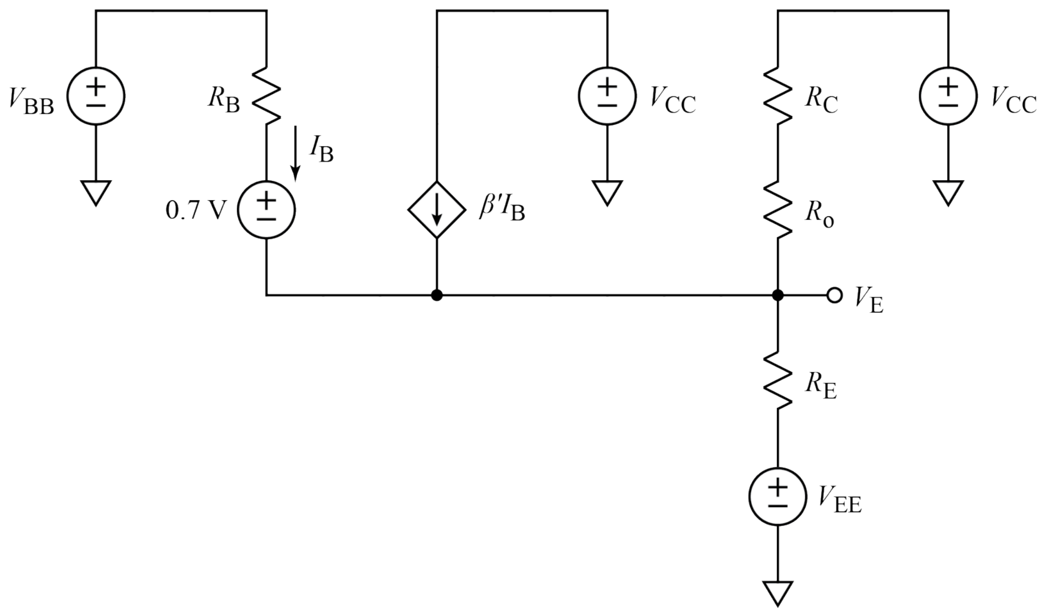


Fig. 4. Collector power supply split into two power supply connections.

All of the circuitry on the far right side of the circuit consists of just resistors and independent sources. Thus, we may convert all of this circuitry into a Thévenin equivalent, denoted as $V_Q$ and $R_Q$, as in Fig. 5

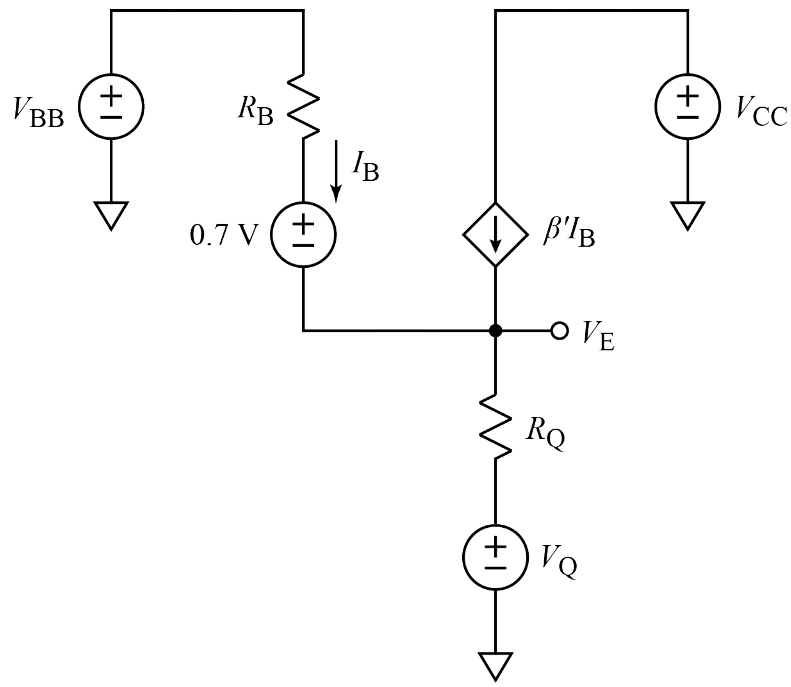


Fig. 5. Emitter and collector power supplies and resistors converted to Thévenin equivalent form. Collector voltage is now absorbed into Thévenin equivalent below the emitter.

Values for the Thévenin equivalent of collector and emitter resistors and power supplies are readily computed using superposition or node-voltage.

$$V_Q = \frac{V_{EE}(R_o + R_C) + V_{CC}R_E}{R_E + R_o + R_C}$$

$$R_Q = R_E \| (R_o + R_C)$$

The circuit in Fig. 5 now looks like a standard DC transistor model with a modified beta and no collector resistance. To find $I_B$, $V_B$, and $V_E$, we use reflected resistance to eliminate the dependent source, as shown in Fig. 6.

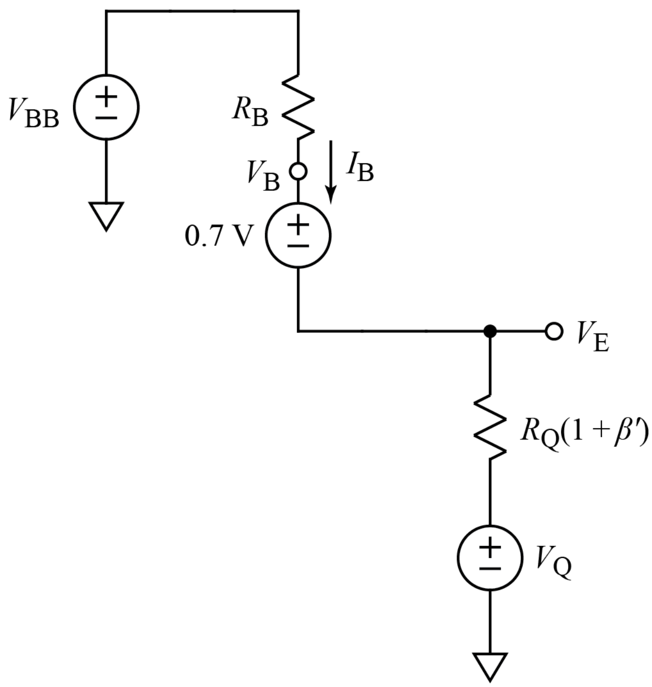


Fig. 6. Impedance multiplication (or reflected resistance) of emitter circuitry yields model with correct $i_B$, $v_B$, and $v_E$ values.

We now apply Kirchhoff's laws and superposition to derive expressions for $I_B$ and $V_E$:

$$I_B = \frac{V_{BB} - 0.7\text{V} - V_Q}{R_B + R_Q(1+\beta')}$$

$$V_E = \frac{(V_{BB} - 0.7\text{V})R_Q(1+\beta') + V_Q R_B}{R_B + R_Q(1+\beta')}$$

The base voltage is just $V_E$ plus 0.7 V, as usual.

$$V_B = \frac{(V_{BB} - 0.7\text{V})R_Q(1+\beta') + V_Q R_B}{R_B + R_Q(1+\beta')} + 0.7\text{ V}$$

or

$$V_B = \frac{(V_{BB} - 0.7\text{V})R_Q(1+\beta') + V_Q R_B}{R_B + R_Q(1+\beta')} + 0.7\text{ V}\frac{R_B + R_Q(1+\beta')}{R_B + R_Q(1+\beta')}$$

or

$$V_B = \frac{(V_{BB})R_Q(1+\beta') + (V_Q + 0.7\text{V})R_B}{R_B + R_Q(1+\beta')}$$

Earlier, in Fig. 2 we transformed the dependent source and $R_o$ to a Thévenin equivalent. Having found $I_B$, we may now treat $R_B$ and the dependent voltage source as known voltage drops, as shown in Fig. 7.

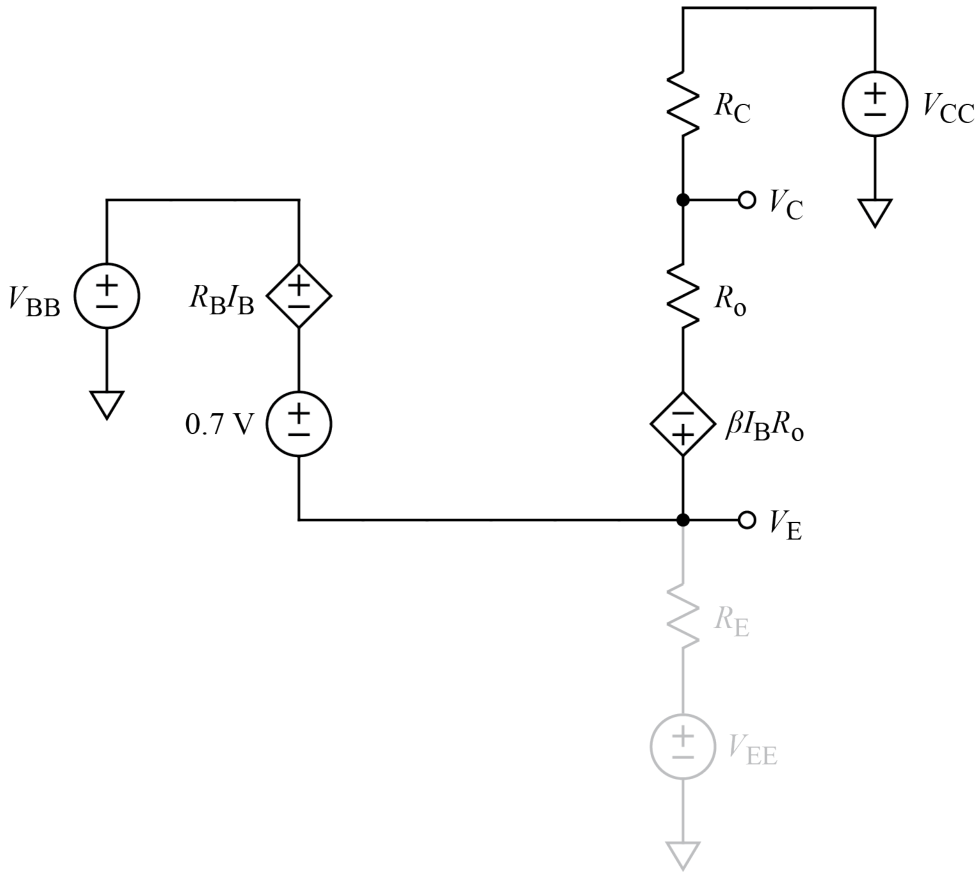


Fig. 7. Model for finding collector voltage, $v_C$, shows base resistor as dependent source so $v_C$ may be computed by superposition of voltage dividers. Dependent current source in transistor is also transformed to Thévenin equivalent form as in Fig. 2. The emitter circuit is grayed out because it has no effect on $V_C$ once $I_B$ is known for the voltage sources.

This model allows us to use a voltage loop around the top part of the circuit, or superposition on the top part of the circuit to determine $V_C$. For convenience, the relevant circuitry is redrawn in Fig. 8.

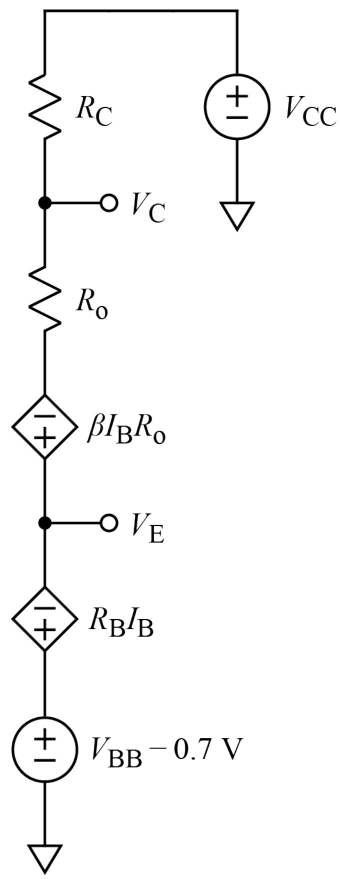


Fig. 8. Model for finding collector voltage, $V_C$, redrawn to emphasize voltage-divider form.

Using superposition, we derive a formula for $V_C$ in terms of $I_B$.

$$V_C = \frac{[(V_{BB} - 0.7\text{V}) - (R_B + \beta R_o)I_B]\, R_C + V_{CC}R_o}{R_o + R_C}$$

where

$$I_B = \frac{V_{BB} - 0.7\text{V} - V_Q}{R_B + R_Q(1 + \beta')}$$

and

$$V_Q = \frac{V_{EE}(R_o + R_C) + V_{CC}R_E}{R_E + R_o + R_C} = \frac{V_{EE}(1 + R_C/R_o) + V_{CC}R_E/R_o}{1 + R_E/R_o + R_C/R_o} \qquad \lim_{R_o \to \infty} V_Q = V_{EE}$$

$$R_Q = R_E \| (R_o + R_C) = \frac{R_E(R_o + R_C)}{R_E + R_o + R_C} = \frac{R_E(1 + R_C/R_o)}{1 + R_E/R_o + R_C/R_o} \qquad \lim_{R_o \to \infty} R_Q = R_E$$

$$\beta' = \frac{\beta R_o}{R_o + R_C} = \frac{\beta}{1 + R_C/R_o} \qquad \lim_{R_o \to \infty} \beta' = \beta$$

Note:

$$R_Q(1+\beta') = \frac{R_E \cancel{(R_o + R_C)}}{R_E + R_o + R_C}\left(\frac{R_o + R_C}{\cancel{R_o + R_C}} + \frac{\beta R_o}{\cancel{R_o + R_C}}\right) = \frac{R_E[R_o(1+\beta) + R_C]}{R_E + R_o + R_C}$$

or

$$R_Q(1+\beta') = \frac{R_E[(1+\beta) + R_C/R_o]}{1 + R_E/R_o + R_C/R_o}.$$

As a consistency check we consider the case of $R_o$ becoming an open circuit, which yields the reflected resistance value expected when there is no $R_o$.

$$\lim_{R_o \to \infty} R_Q(1+\beta') = R_E(1+\beta) \quad ✔$$

We get the expected result.