TOOL: The transformations below yield an expression for the gain of a BJT in the AC case for one transistor, shown in Fig. 1.

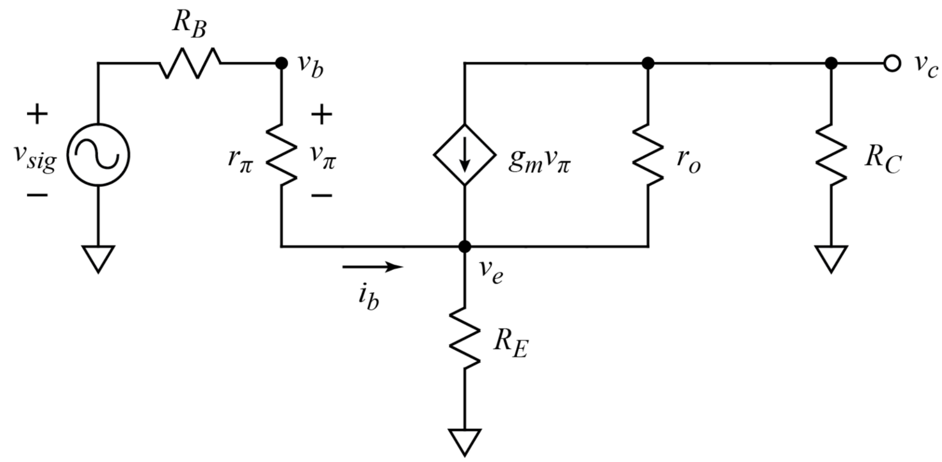

Fig. 1. Generic model for one-transistor circuit with Early resistance.

Early resistance for AC is given by a formula in terms of Early voltage (from datasheet) for the transistor and DC current from collector to emitter.

$$r_o = \frac{|V_A|}{I_C}$$

To find base current, i_b, we first transform the dependent source and r_o into a Thévenin equivalent, (see Fig. 2).

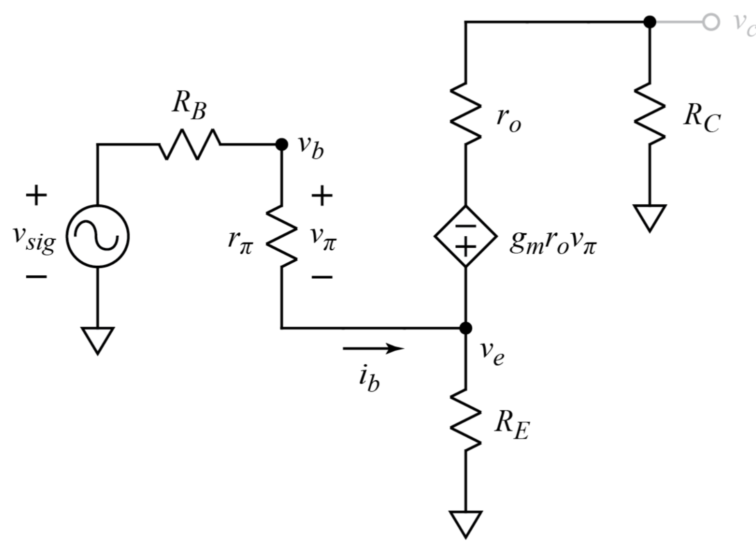

Fig. 2. Dependent source transformed to Thévenin equivalent.

Combining the two resistors in the collector, we transform back into a Norton equivalent, as shown in Fig. 3.

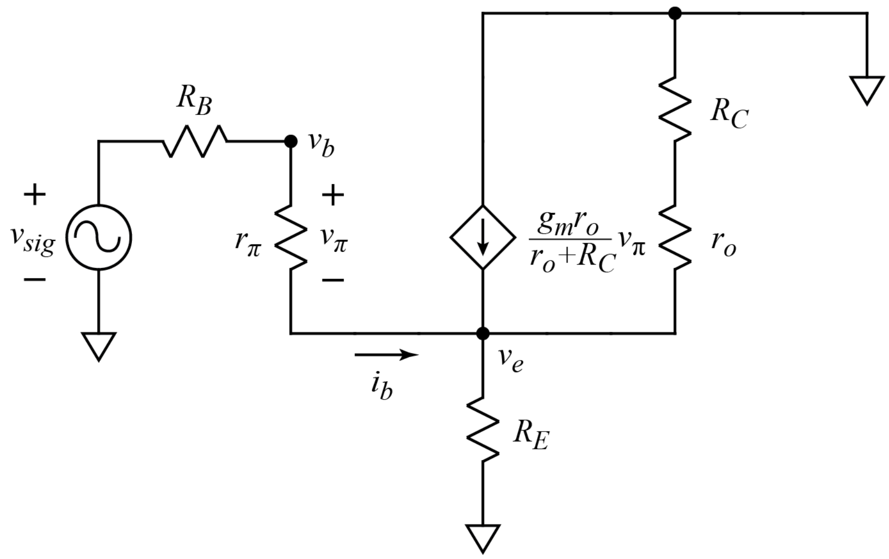

Fig. 3. Thévenin form including R_C equivalent transformed to Norton equivalent form.

We now define a modified g_m value for the transistor and split the collector reference connection in two, as shown in Fig. 4.

$$g_m' = \frac{g_m r_o}{r_o + R_C}$$

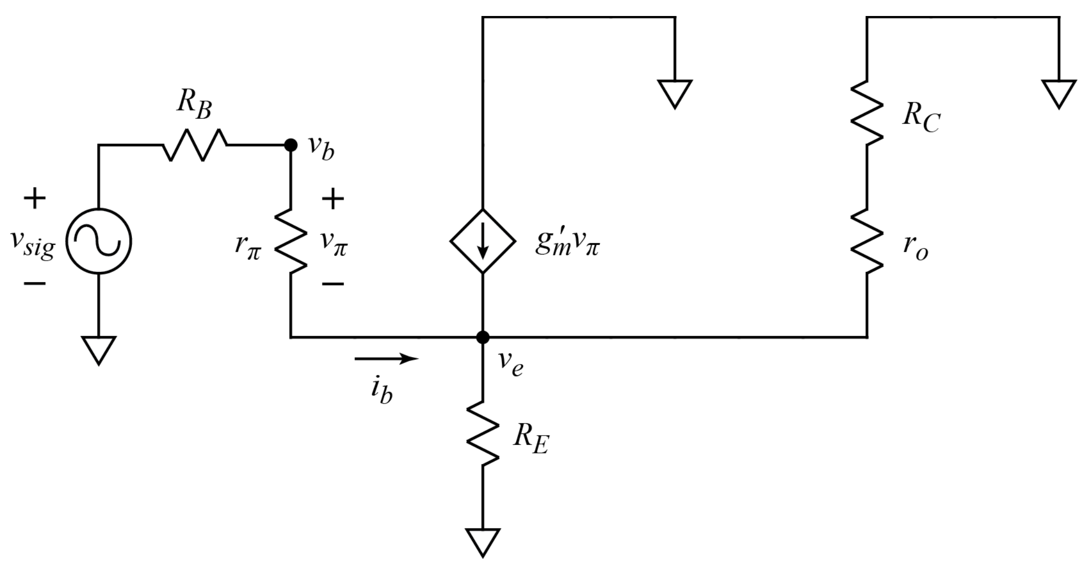

Fig. 4. Collector reference connection split into two separate reference connections.

It now becomes apparent that the collector and emitter resistors, as shown in Fig. 5, are in parallel and may be combined.

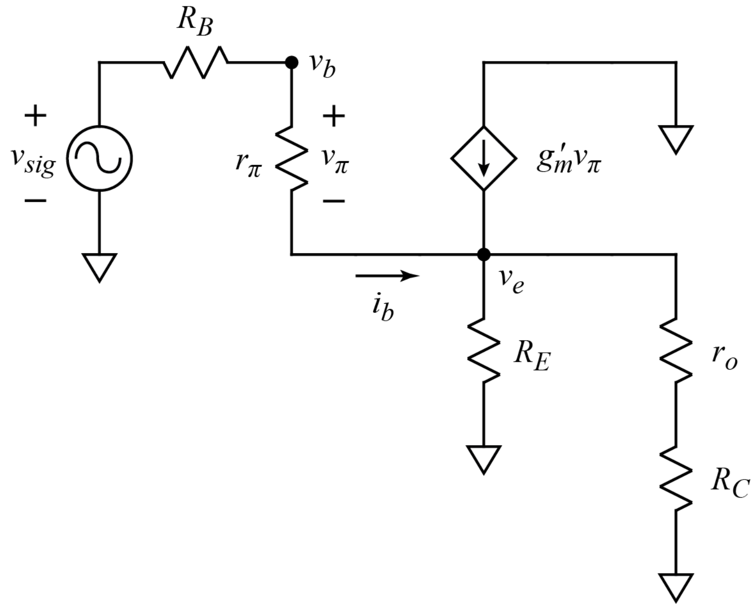

Fig. 5. Emitter and collector resistors are now in parallel in emitter circuit. Collector voltage absorbed into Thévenin equivalent is now absent, but the model may be used to calculate i_b.

The equivalent of collector and emitter resistors is now denoted as R_q in Fig. 6.

$$R_q = R_E \| (r_o + R_C)$$

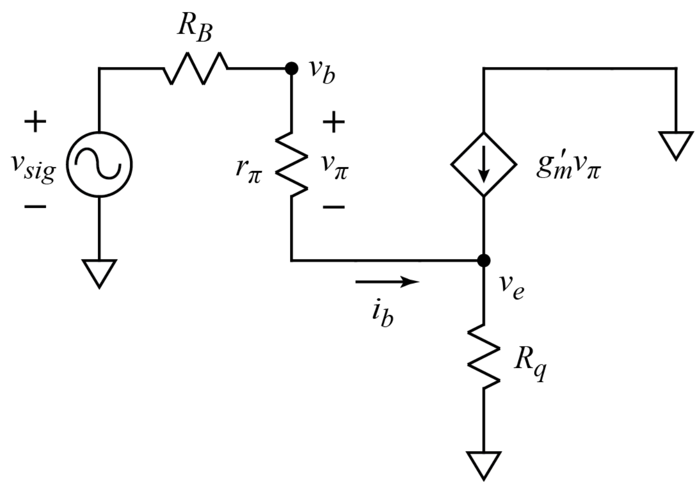

Fig. 6. Equivalent resistance, R_q, in emitter circuit.

Using the concept of reflected resistance, we eliminate the dependent source, as shown in Fig. 7.

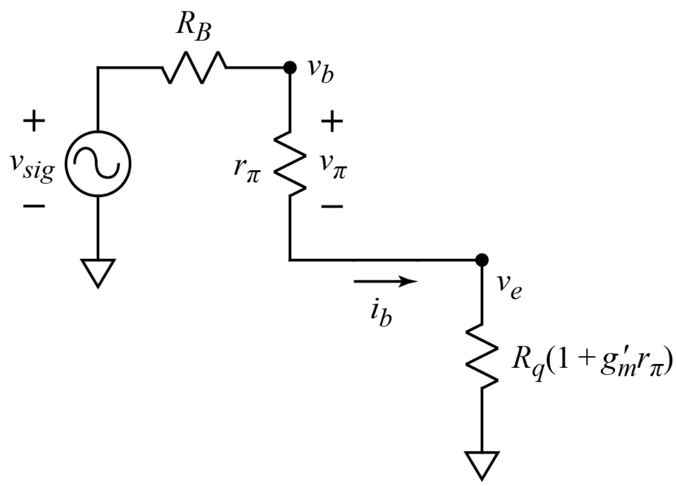

Fig. 7. Impedance multiplication (or reflected resistance) of emitter circuitry yields model with correct i_b, v_π, and v_E values.

We now apply Kirchhoff's laws and superposition to derive expressions for i_b, v_π, and v_e:

$$i_B = \frac{v_{\text{sig}}}{R_B + r_\pi + R_q(1 + g_m' r_\pi)}$$

$$v_\pi = v_{\text{sig}} \frac{r_\pi}{R_B + r_\pi + R_q(1 + g_m' r_\pi)}$$

$$v_E = v_{\text{sig}} \frac{R_q(1 + g_m')}{R_B + r_\pi + R_q(1 + g_m' r_\pi)}$$

where

$$R_q = R_E \| (r_o + R_C)$$

$$g_m' = \frac{g_m r_o}{r_o + R_C}$$

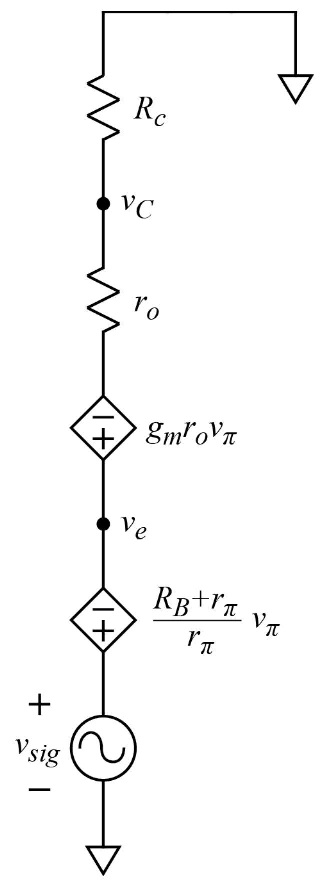

Fig. 8. Model for finding collector voltage, v_C, shows base resistor as dependent source so v_C may be computed by superposition of voltage dividers. Dependent current source in transistor also transformed to Thévenin equivalent form as in Fig. 2.

Using superposition and the earlier formula for v_π, we derive a formula for v_C:

$$v_C = \left[v_{\text{sig}} - v_\pi \frac{R_B + r_\pi}{r_\pi} - v_\pi g_m r_o \right] \frac{R_C}{r_o + R_C}$$

or

$$v_C = \left[v_{\text{sig}} - v_\pi \left(\frac{R_B + r_\pi}{r_\pi} + \frac{g_m r_o r_\pi}{r_\pi} \right) \right] \frac{R_C}{r_o + R_C}$$

or

$$v_C = v_{\text{sig}} \left[1 - \frac{r_\pi}{R_B + r_\pi + R_q(1 + g_m' r_\pi)} \frac{R_B + r_\pi + g_m r_o r_\pi}{r_\pi} \right] \frac{R_C}{r_o + R_C}$$

or

$$v_C = v_{\text{sig}}\left[1 - \frac{R_B + r_\pi + g_m r_o r_\pi}{R_B + r_\pi + R_q(1 + g'_m r_\pi)}\right]\frac{R_C}{r_o + R_C}$$

or

$$v_C = v_{\text{sig}}\left[\frac{R_q(1 + g'_m r_\pi) - g_m r_o r_\pi}{R_B + r_\pi + R_q(1 + g'_m r_\pi)}\right]\frac{R_C}{r_o + R_C}$$

If the output is taken as V_C, the gain of the circuit is

$$A_v \equiv \frac{v_C}{v_{sig}} = \frac{R_q(1+g'_m r_\pi) - g_m r_o r_\pi}{R_B + r_\pi + R_q(1+g'_m r_\pi)} \cdot \frac{R_C}{r_o + R_C}.$$

Assuming there is a bias network creating a resistance of R_{Bb} in parallel with v_{sig}, the R_{in} of the circuit is given by R_{Bb} in parallel with the equivalent resistance of the base current, which is v_{sig}/i_B. Using the formula for i_B from above, we have the input resistance:

$$R_{in} = R_{Bb} \| [R_B + r_\pi + R_q(1 + g'_m r_\pi)].$$

For the output resistance, R_{out}, see separate Conceptual Tools page.