TOOL: The transformations presented here yield expressions for input resistance, R_{in}, output resistance, R_{out}, and voltage gain, A_v of a Common Base (CB) BJT circuit with Early resistance, r_o. Fig. 1 shows the circuit.

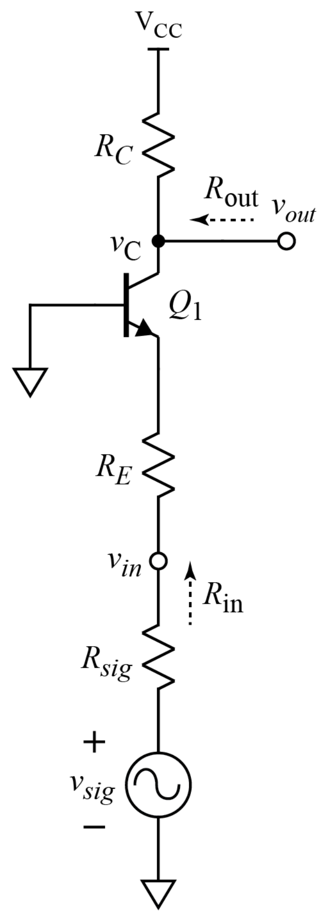

Fig. 1. CE amplifier with emitter degeneration.

Fig 2 shows the AC model of the CB amplifier where the DC sources are turned off (they become wires). Resistor r_o represents the Early effect. Early resistance for AC is given by a formula in terms of Early voltage from the datasheet for the transistor and DC collector current.

$$r_o = \frac{|V_A|}{I_C}$$

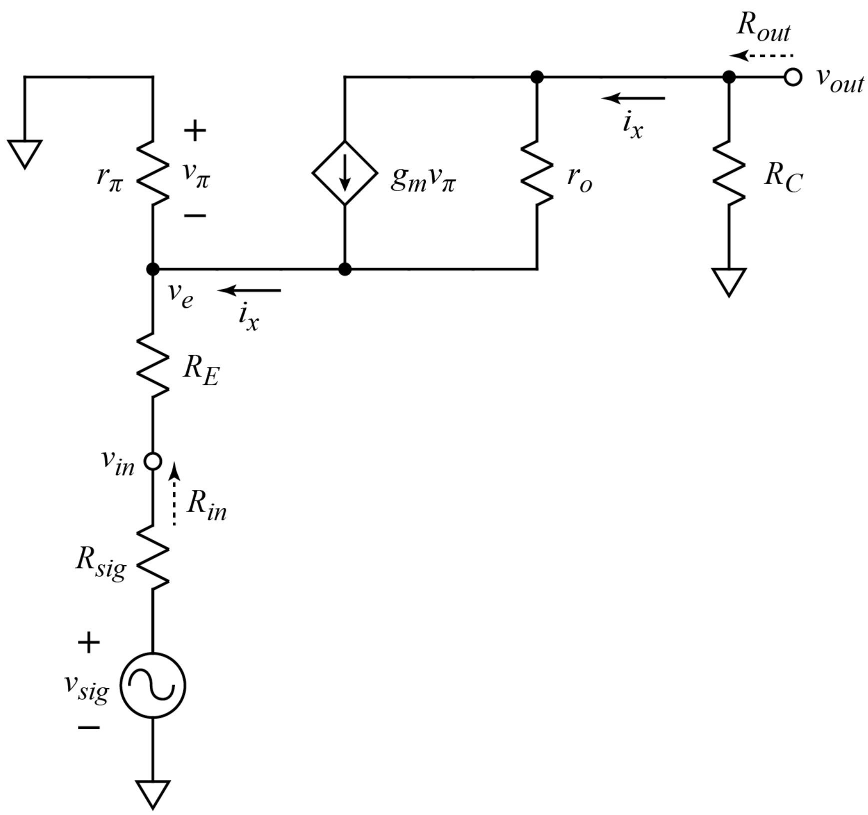

Fig. 2. CB amplifier AC model.

To calculate the input resistance, a 1V source is connected to the input, as shown in Fig. 3, and the input current, i_{in}, is determined. Applying Ohms law gives the value of input resistance.

$$R_{in} = \frac{1\text{V}}{i_{in}}$$

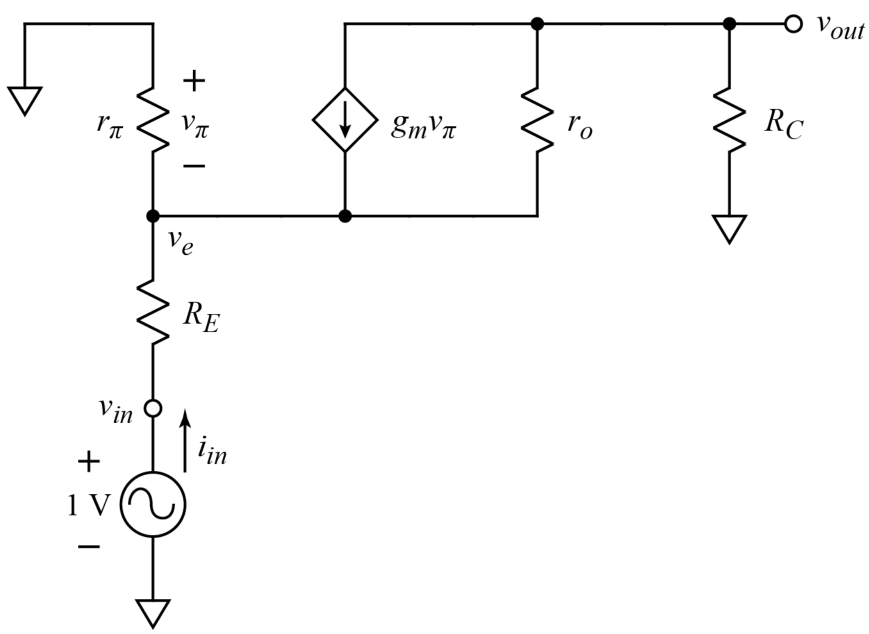

Fig. 3. AC model for calculation of R_{in}.

However, we simplify the problem by moving the 1V source inside the emitter resistance, R_E, and adding the R_E to input resistance later. Fig. 4 shows the circuit.

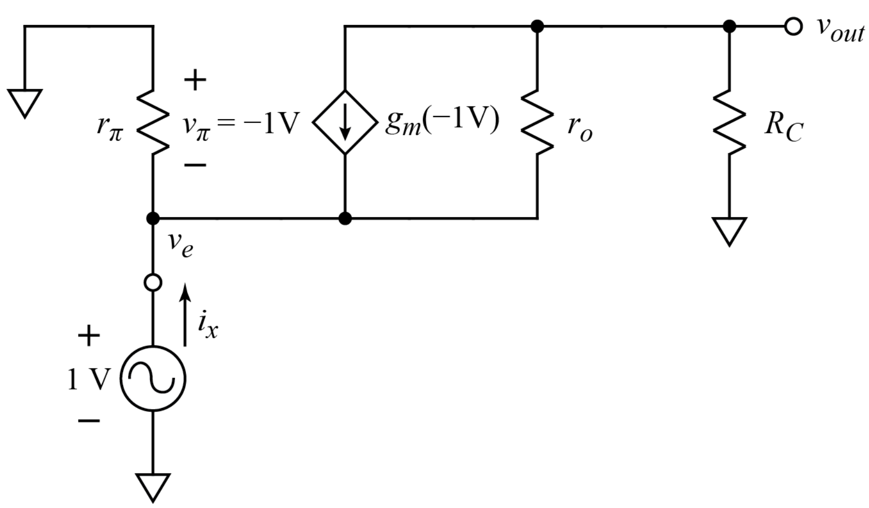

Fig. 4. AC model with input moved inside R_E.

With the 1V source connected at the emitter, the voltage across the incremental resistance, r_π, is −1V, and the dependent current source value is known. We may convert the current source and parallel r_o into a Thévenin equivalent. Fig. 5 shows the new model.

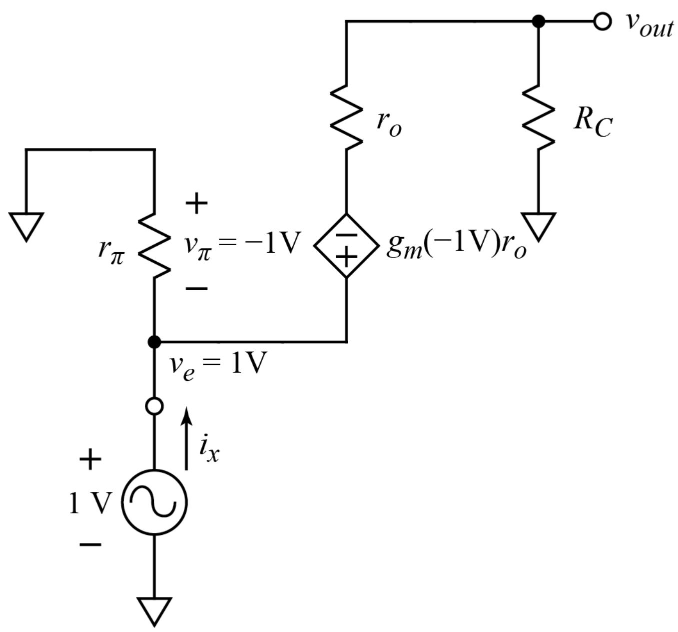

Fig. 5. AC model with Thévenin equivalent for calculation of R_{in}.

The total current, i_{in}, flowing into the circuit may now be written as a formula.

$$i_x = \frac{1\text{V}}{r_\pi} + \frac{1\text{V} - g_m(-1\text{V})r_o)}{r_o + R_C} = \frac{1\text{V}}{r_\pi} + \frac{1\text{V}(1 + g_m r_o)}{r_o + R_C}$$

Applying Ohm's law and adding R_E yields the formula for R_{in}.

$$R_{in} = \frac{1\text{V}}{i_x} + R_E = \frac{1\text{V}}{\dfrac{1\text{V}}{r_\pi} + \dfrac{1\text{V}(1 + g_m r_o)}{r_o + R_C}} + R_E = r_\pi || \frac{r_o + R_C}{1 + g_m r_o} + R_E$$

NOTE: When this amplifier block is connected to another block, the R_{in} value is modified by replacing R_C with R_C in parallel with the input resistance of the next amplifier block.

The output resistance, R_{out}, is the Thévenin equivalent resistance of the AC model as seen from the output of the circuit. To calculate the output resistance, the input is turned off (i.e., connected to reference), and a 1V source is connected to the output, as shown in Fig. 6. Then the input current, i_{in}, is determined, and we calculate the output resistance from Ohm's law.

$$R_{out} = \frac{1\text{V}}{i_{in}}$$

The key ideas for the derivation of the output resistance are to first observe that the 1V is present to the left of R_C, (so we may add R_C in parallel with the output resistance to the left of R_C), and to second observe that the current i_x flowing into the collector of the transistor also flows into the junction between r_π and R_E.

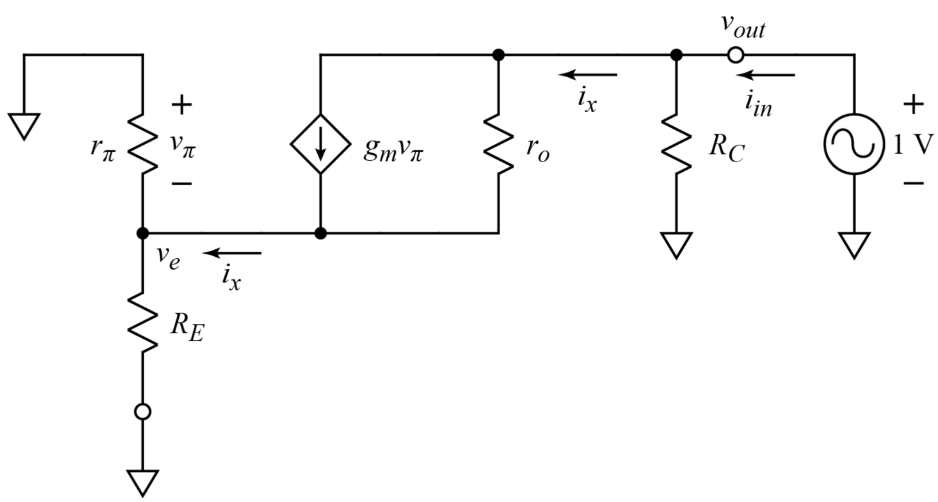

Fig. 6. AC model with Thévenin equivalent for calculation of R_{out}.

Fig. 7 shows the circuit model for calculating i_x. without R_C.

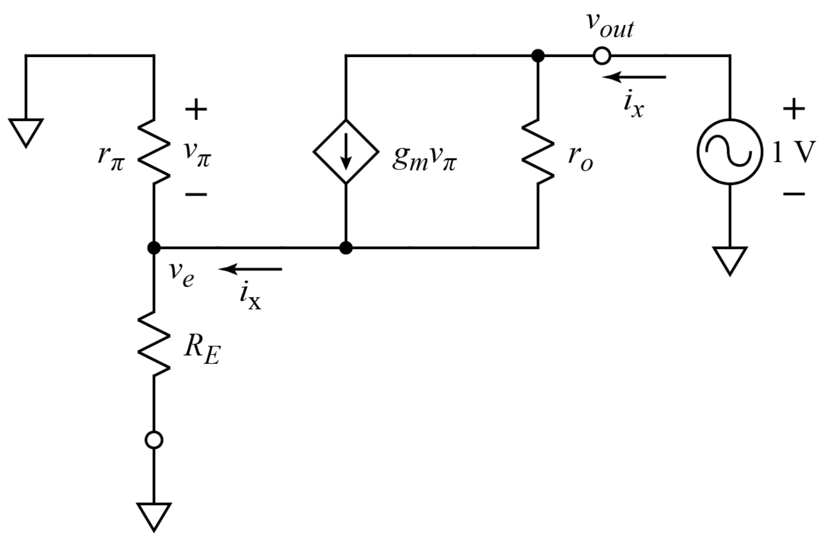

Fig. 7. AC model for finding i_x.

We may convert the dependent source and r_o to a Norton equivalent, as Fig. 8 shows. If we consider i_x, we find that, on the left side, we have v_π across r_π and R_E and may express i_x as $v_\pi/r_\pi||R_E$.

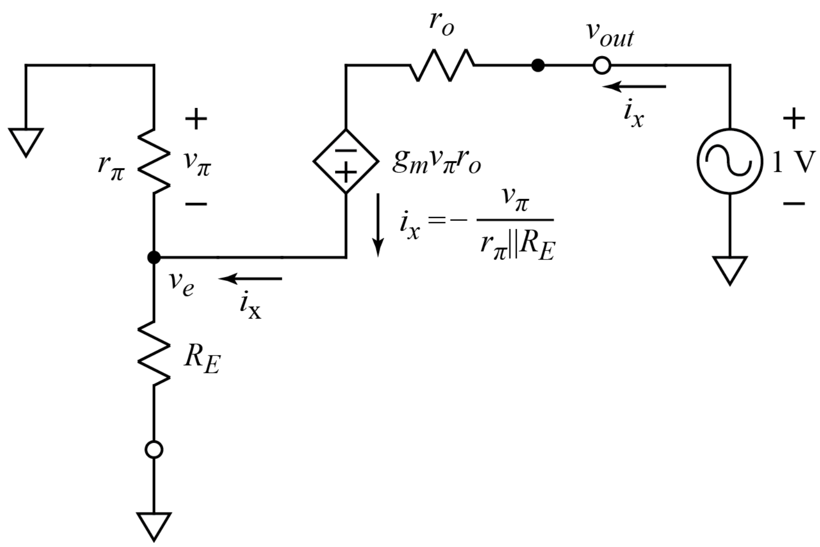

Fig. 8. Dependent source converted to Thévenin equivalent and i_x expressed in terms of v_π.

Writing i_x in terms of v_π allows us to replace the dependent source with an equivalent resistance by employing Ohm's law and cancelling v_π.

$$R_{eq} = \frac{v}{i} = \frac{g_m v_\pi r_o}{\dfrac{v_\pi}{r_\pi || R_E}} = g_m r_o \cdot r_\pi || R_E$$

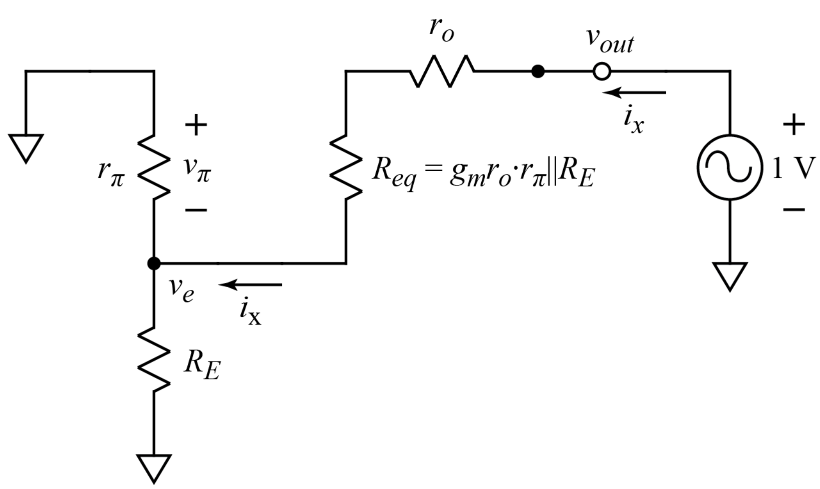

Fig. 9.

Now we add the R_C back into the picture, as shown in Fig. 10.

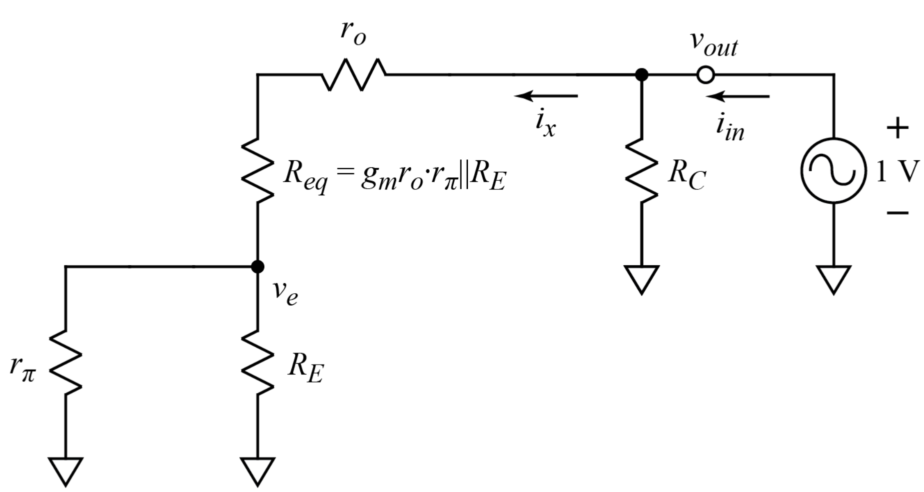

Fig. 10.

Based on Fig. 10, we may directly write down the output resistance.

$$R_{out} = \frac{1\text{V}}{i_{in}} = R_C||(r_\pi||R_E + g_m r_o \cdot r_\pi||R_E + r_o) = R_C||[(1+g_m r_o)(r_\pi||R_E)]$$

NOTE: When this amplifier block is connected to another block, the R_{out} value is modified by replacing R_E with R_E in series with the output resistance of the next amplifier block.

To find the voltage gain, $A_v = v_{out}/v_{in}$, we use input signal v_{in} (without R_{sig}) and create a voltage divider to find v_e by recalling Fig. 5 and treating the circuit above R_E as the resistance (now denoted as $R_{in,e}$) found earlier in Fig. 5. Fig. 11 shows the model.

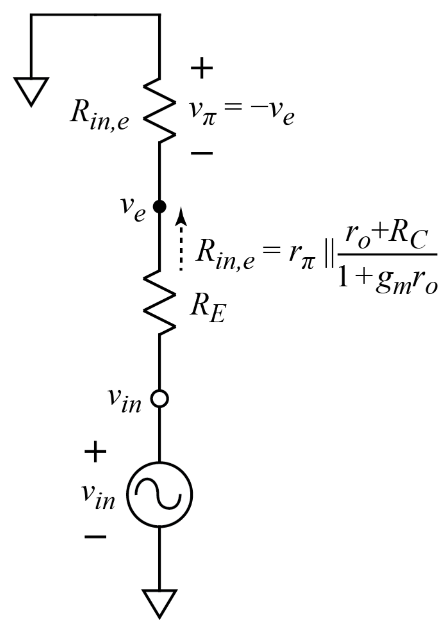

Fig. 11. Voltage-divider model for finding v_e.

The voltage divider formula for emitter voltage is

$$v_e = \frac{R_{in,e}}{R_E + R_{in,e}}$$

where

$$R_{in,e} = r_\pi||\frac{r_o + R_C}{1 + g_m r_o}.$$

Now that v_e is known, we use the model in Fig. 12 that is Fig. 5 adapted to use v_e.

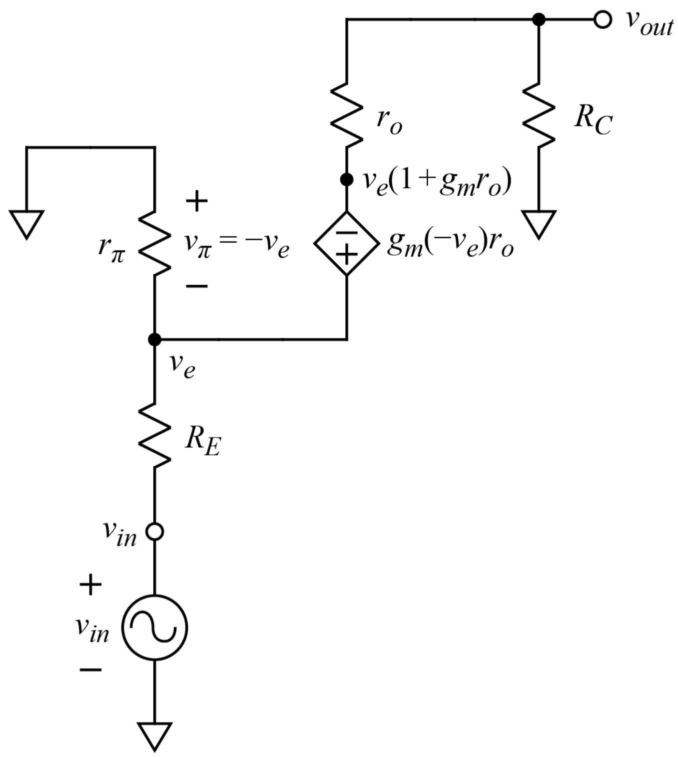

Fig. 12.

We have a voltage divider for vout.

$$v_{out} = v_{in} \frac{R_{in,e}}{R_E + R_{in,e}} (1 + g_m r_o) \frac{R_C}{R_C + r_o}$$

Now we have the gain formula.

$$A_v = \frac{v_{out}}{v_{in}} = \frac{R_{in,e}}{R_E + R_{in,e}} (1 + g_m r_o) \frac{R_C}{R_C + r_o}$$

where

$$R_{in,e} = r_\pi || \frac{r_o + R_C}{1 + g_m r_o}.$$

NOTE: When this amplifier block is connected to another block, the $R_{in,e}$ value is modified by replacing R_C with R_C in parallel with the input resistance of the next amplifier block. (Another way to deal with this issue is to convert results to a 2-port model.)

Fig 13 shows the generic amplifier model for which we have found values.

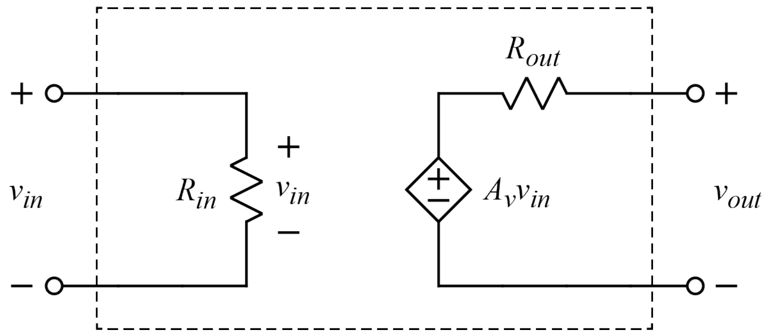

Fig. 13. Generic amplifier model

REF: Behzad Razavi, *Fundamentals of Microelectronics with Robotics and Bioengineering Applications, 3rd Ed., Enhanced eText,* New York, NY: Wiley, 2021. ISBN: 9781119694397