TOOL: The transformations presented here yield an expression for the AC output resistance, R_{out}, of a Common Emitter (CE) BJT circuit with emitter degeneration and Early resistance. Fig. 1 shows the circuit.

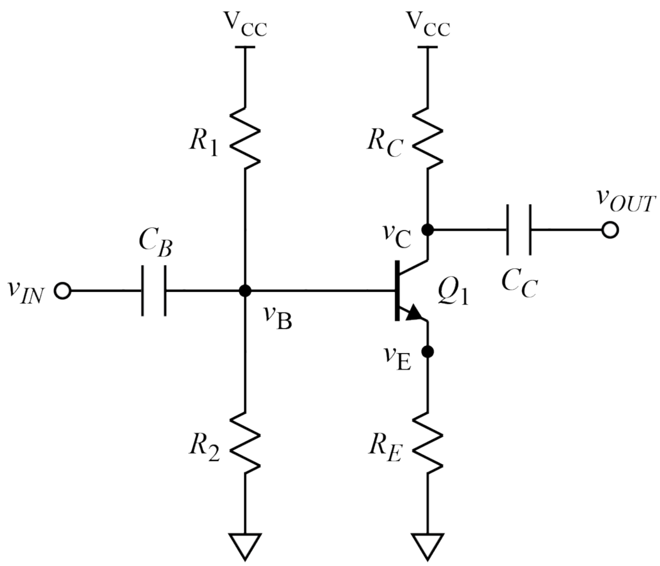

Fig. 1. CE amplifier with emitter degeneration.

Fig 2 shows the AC model of the CE amplifier. The DC sources and AC input are turned off (they become wires), the capacitors are treated as short circuits for AC (they also become wires), and R_B is the parallel resistance of the output resistance, R_{sig}, of the circuit driving v_{in} and the bias resistors R_1 and R_2, i.e. $R_B = R_{sig} \parallel R_1 \parallel R_2$. ($R_B$ is often taken to be zero.) Resistor r_o represents the Early effect.

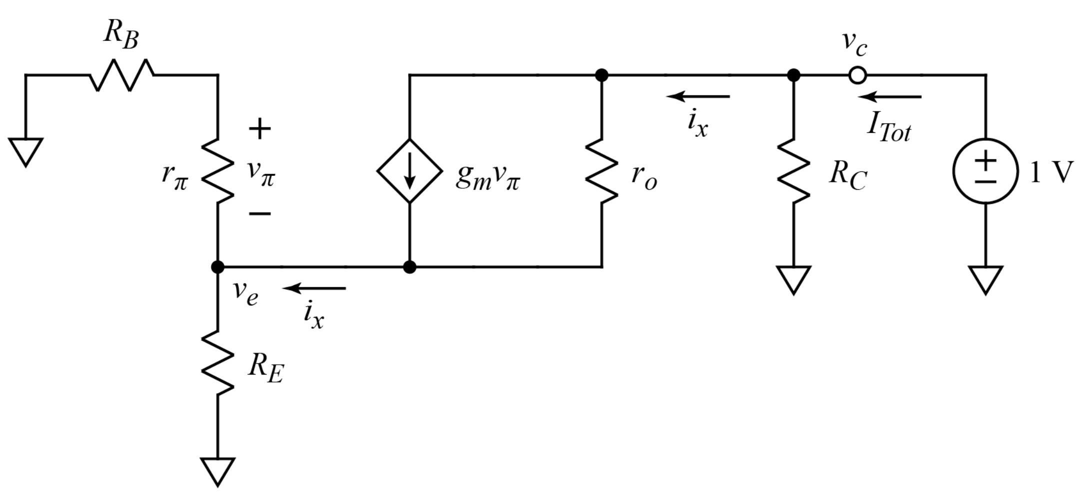

Fig. 2. AC model for circuit.

Early resistance for AC is given by a formula in terms of Early voltage from the datasheet for the transistor and DC collector current.

$$r_o = \frac{|V_A|}{I_C}$$

The output resistance is the Thévenin equivalent resistance of the AC model as seen from the output of the circuit. The key ideas for the derivation of the output resistance are to observe that current into the collector is the same as the sum of currents flowing out of the emitter and base, that the emitter and base current are a simple function of the emitter voltage, and the emitter voltage may be found from a reflected resistance model. The following steps lead to the formula for R_{out}:

Summary of Steps

1) Turn off the input source and attach a 1V (AC) source to the output, as shown in Fig. 3.
2) Remove R_C and put it in parallel with the output resistance of the remaining circuit. Note that i_x is found in two locations, as shown in Figs. 3 and 4.
3) The i_x flowing into v_E is given by v_E and the values of R_B, R_π, and R_E.
4) We can find v_E by taking the Thévenin equivalent of r_o and R_E.
5) With the Thévenin equivalent from (4), we can use the idea of reflected resistance to obtain a simple model for v_E.
6) Having obtained v_E, we calculate i_x, and use $R_{out} = 1\text{V}/i_x$.

Details of Steps

Fig. 3 illustrates Step 1 of the process.

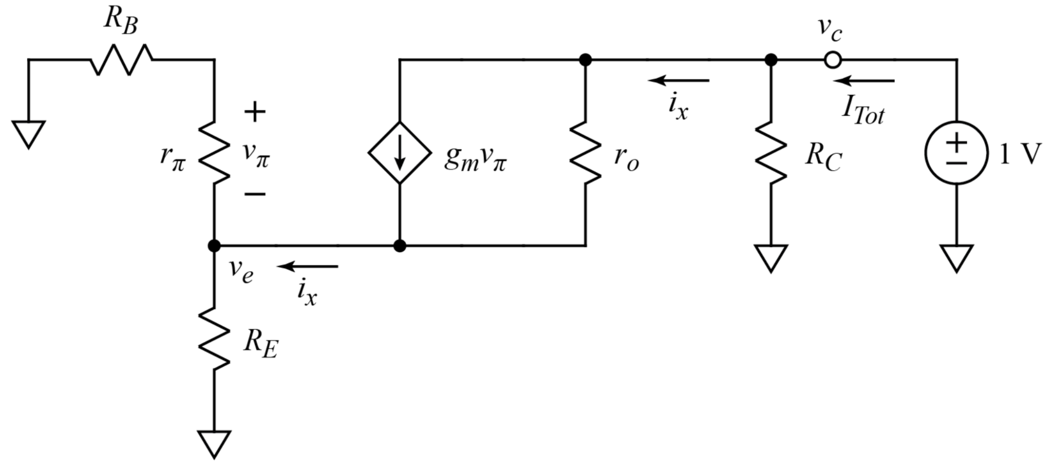

Fig. 3. Setup for determining output resistance for one-transistor CE amplifier circuit.

The collector resistor, R_C, is removed in Step 2, as shown in Fig. 4. At the end, we put R_C in parallel with the output resistance of the circuit in Fig. 4. Now we need only compute i_x. We observe that i_x appears in two locations in the circuit.

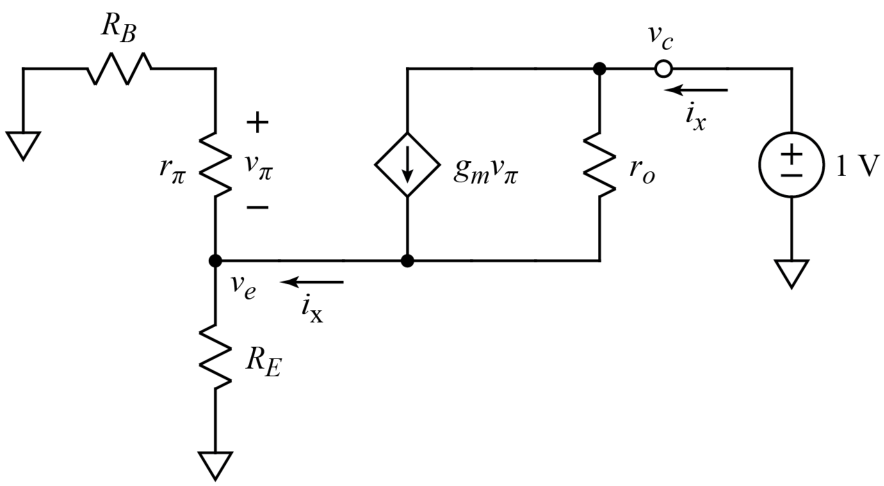

Fig. 4. Generic model for one-transistor Common Emitter (CE) circuit.

Step 3 is to observe that i_x may be easily computed as v_e divided by the parallel resistance $(R_B + r_\pi) \parallel R_E$. The focus now is on finding a way to compute v_e.

At step 4, things take a surprising turn, in that we lose one of the i_x values in a transformation that makes a calculation of v_e possible. The idea is that r_o and R_E form a voltage divider across the 1 V source, as emphasized by redrawing the circuit as in Fig. 5.

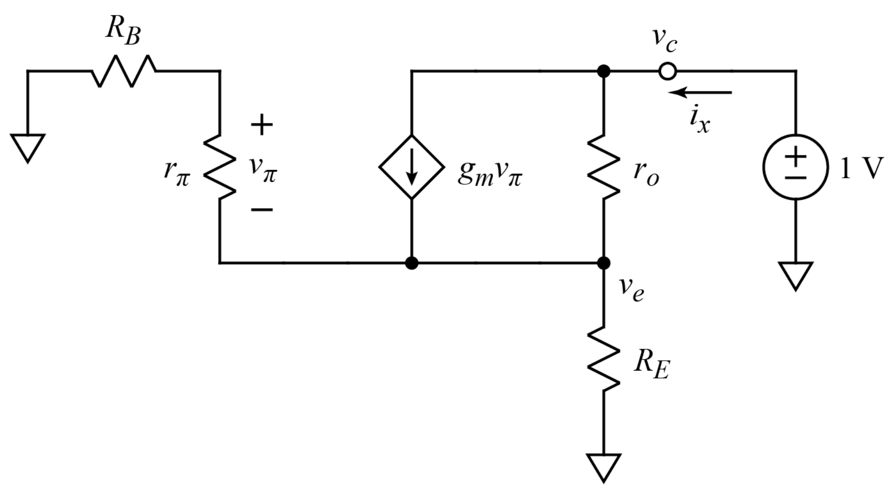

Fig. 5. AC circuit model emphasizing that r_o and R_E form a voltage divider.

Fig. 6 takes things further by separating the dependent source and voltage divider. The other side of the current source is now connected to reference.

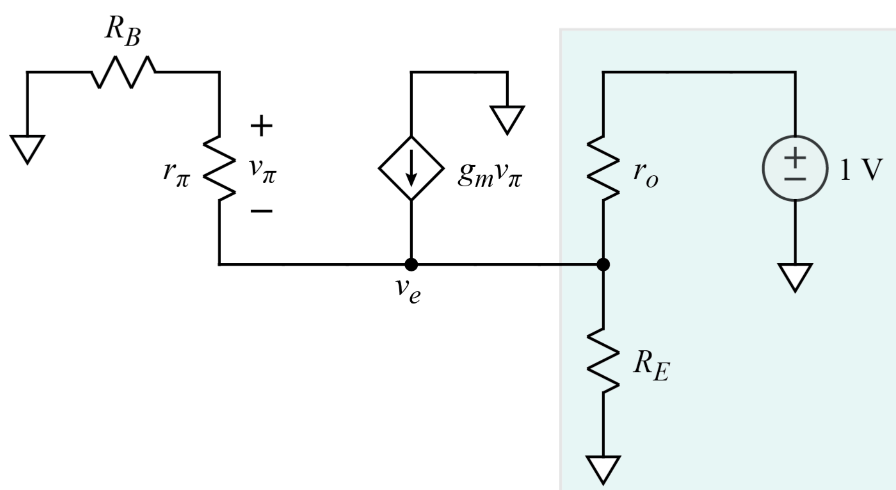

Fig. 6. AC circuit model with dependent source separated from voltage divider formed by 1V source, r_o, and R_E.

Since we are creating a model to find v_e and may ignore the value of the total current flowing out of the current source, connecting the other side of the dependent source to reference is allowed. Note that we could equally well have connected the dependent current source to its own 1V source if we wanted to preserve some representation of i_x. However, we are only interested in v_e, at present. Fig. 6 is slightly less cluttered and still gives the correct value of v_e.

In Fig. 7, we have transformed the voltage divider into its Thévenin equivalent. The circuit now takes on a familiar form to which the concept of reflected resistance applies.

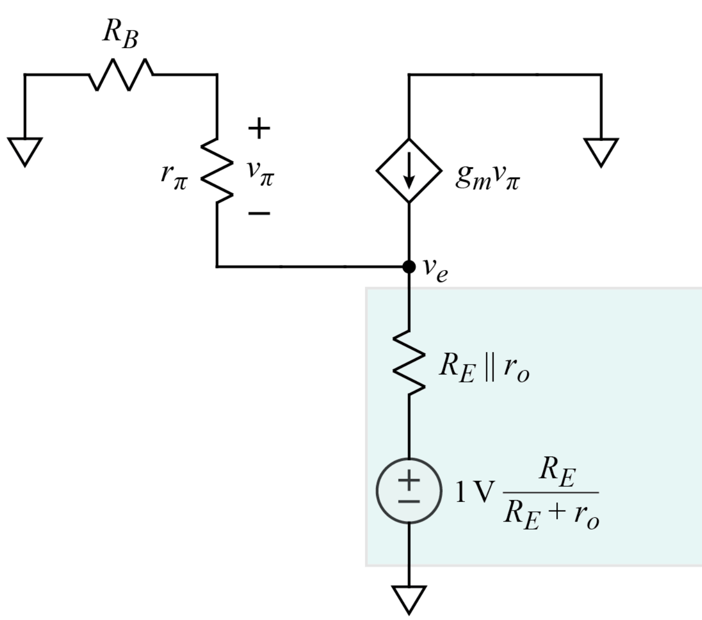

Fig. 7. Circuit with Thévenin equivalent of 1V source, r_o, and R_E.

Fig. 8 shows the circuit transformed into the reflected resistance model. Current i_x has long since been transformed away, but v_e has the correct value. The circuit is now a voltage divider.

$$v_e = 1\text{ V}\frac{R_E}{R_E + r_o} \cdot \frac{\dfrac{R_B + r_\pi}{1 + g_m r_\pi}}{\dfrac{R_B + r_\pi}{1 + g_m r_\pi} + R_E \| r_o} = \frac{1\text{ V}}{R_E \| r_o}\frac{R_E}{R_E + r_o} \cdot \frac{R_B + r_\pi}{1 + g_m r_\pi}\|R_E\|r_o$$

Note that the || operator binds more tightly than the multiplications. The term in front simplifies to $1/r_o$, giving a more compact expression for v_e.

$$v_e = 1\text{ V}\frac{1}{r_o} \cdot \frac{R_B + r_\pi}{1 + g_m r_\pi}\|R_E\|r_o$$

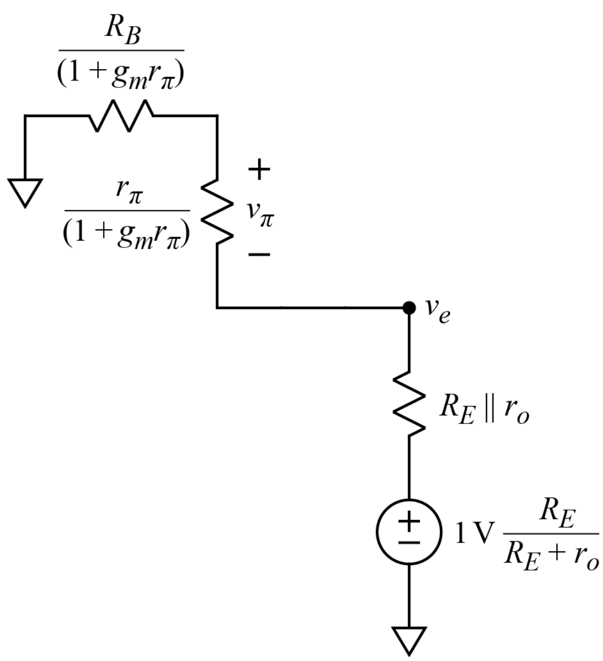

Fig. 8. Reflected resistance model of circuit. The model yields the correct value v_e.

Now we are able to compute i_x.

$$i_x = \frac{v_e}{(R_B + r_\pi)\|R_E}$$

or

$$i_x = \frac{v_e}{(R_B + r_\pi)||R_E} = 1\text{ V}\frac{1}{r_o} \cdot \frac{\dfrac{R_B + r_\pi}{1 + g_m r_\pi}||R_E||r_o}{(R_B + r_\pi)||R_E}$$

Finally, we have our computation of the output resistance for the circuit without R_C.

$$R_x = \frac{1\text{ V}}{i_x} = r_o \frac{(R_B + r_\pi)||R_E}{\dfrac{R_B + r_\pi}{1 + g_m r_\pi}||R_E||r_o},$$

and with R_C:

$$R_{out} = R_x||R_C.$$

The above result may be obtained more directly using a Norton form for the voltage divider, as shown in Fig. 9, followed by using reflected resistance, as shown in Fig. 10. The earlier result for ix follows directly from the model in Fig. 10.

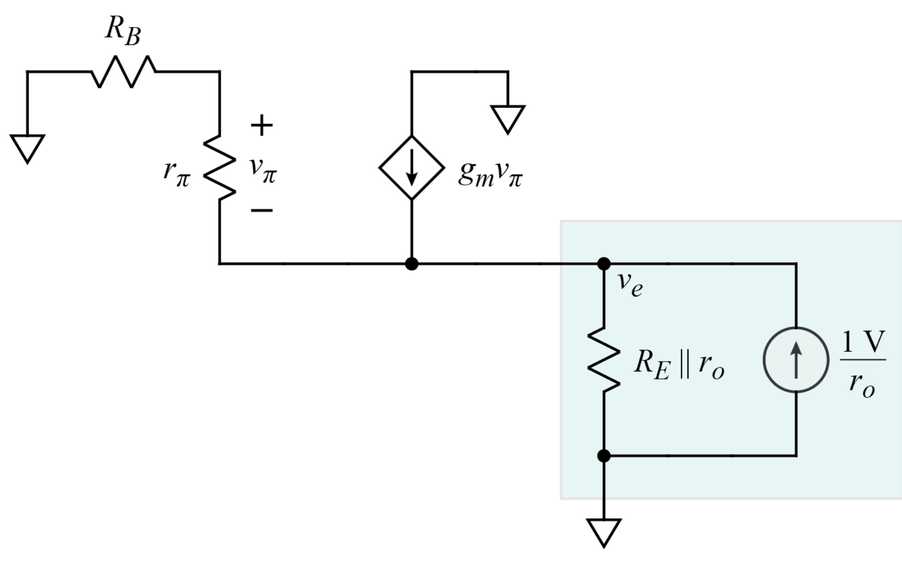

Fig. 9. Norton equivalent substituted for voltage divider.

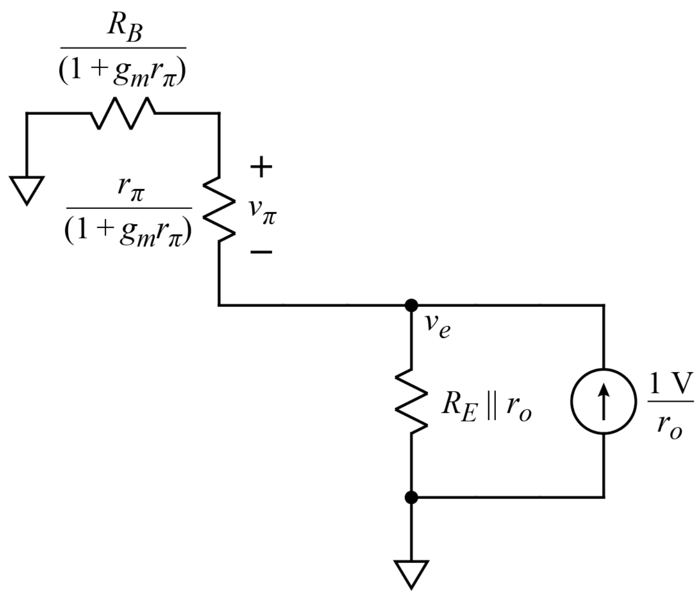

Fig. 10. Norton equivalent and reflected impedance model.

For a consistency check as r_o approaches infinity, the expression for R_x takes on an interesting intermediate form:

$$R_x \approx r_o \frac{(R_B + r_\pi)\|R_E}{\frac{R_B + r_\pi}{1 + g_m r_\pi}\|R_E} = r_o \frac{R_B + r_\pi + (1 + g_m r_\pi)R_E}{R_B + r_\pi + R_E}$$

or

$$R_x \approx r_o \left[1 + g_m r_\pi \frac{R_E}{R_B + r_\pi + R_E}\right]$$

For the typical case of emitter degeneration, where $R_E \gg R_B + r_\pi$, we have something similar to reflected resistance.

$$R_x \approx r_o \left[1 + g_m r_\pi\right]$$

Thus, emitter degeneration has the desirable benefit of effectively increasing r_o. Conversely, if $R_E = 0$, we get just r_o.

Finally, as r_o approaches infinity, we find that R_x approaches infinity and disappears from the circuit model, as expected.

REF: Behzad Razavi, *Fundamentals of Microelectronics with Robotics and Bioengineering Applications, 3rd Ed., Enhanced eText,* New York, NY: Wiley, 2021. ISBN: 9781119694397