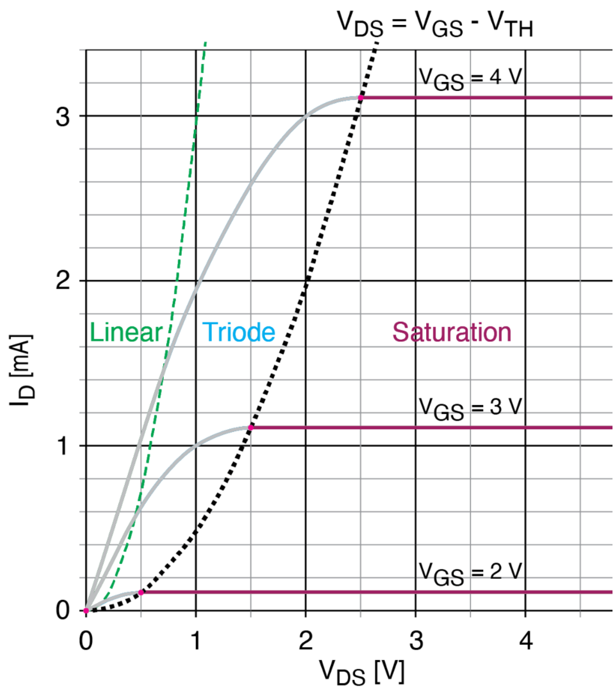
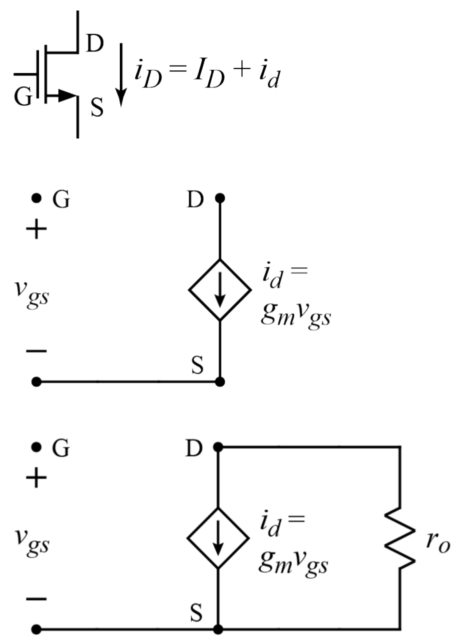
EX: Consider an NMOS FET with channel modulation.

Transconductance parameter	k_n	1 mA/V^2
Threshold	V_{TH}	1.5 V
Channel modulation	λ	0.02 V^{-1}





THEORY: AC models describe the change in Drain current, i_D, for small changes in v_{GS} and/or v_{DS}, [1, 2]. The small changes in voltages allow us to argue that squared and higher-order terms are small, so we may linearize the i_D formula by taking a one- or two-dimensional Taylor series around a DC operating point and truncating it after the linear term.

$$i_D(V_{GS}+v_{gs}, V_{DS}+v_{ds}) \approx i_D(V_{GS}, V_{DS}) + \frac{di_D(dV_{GS}, V_{DS})}{dV_{GS}} v_{gs} + \frac{di_D(V_{GS}, V_{DS})}{dV_{DS}} v_{ds}$$

We model the small (AC) changes in i_D using a current source from drain to source, $g_m v_{gs}$ and, if needed, a resistor, r_o, parallel the current source. We refer to g_m as the "transconductance" and r_o as the channel-modulation resistance.

$$i_D(V_{GS}+v_{gs}, V_{DS}+v_{ds}) \approx i_D(V_{GS}, V_{DS}) + g_m v_{gs} + v_{ds}/r_o$$

Channel modulation, $\lambda \neq 0$, causes the characteristic curves to be sloped in the saturation region to the right of the curve where $V_{DS} = V_{GS} - V_{TH}$. The slope may be modeled by a resistor, r_o, in parallel with the current source in the AC models.

According to one question and response online about channel modulation in the triode and linear regions, the formula for drain current still includes the $(1+\lambda V_{DS})$ multiplier term [3, 4]. This makes sense, as it yields smooth curves.

MODEL: (Sat) Without channel modulation: $I_D = \frac{k_n}{2}(V_{GS}-V_{TH})^2$

$$i_d = g_m v_{gs}$$

$$g_m = \frac{dI_D}{dV_{GS}} = k_n(V_{GS}-V_{TH}) = \frac{2I_D}{V_{GS}-V_{TH}} = \sqrt{2\mu_n C_{ox}\frac{W}{L}I_D}$$

$r_o = \infty\,\Omega$ (omitted from schematic)

With channel modulation: $I_{Dmod} = \frac{k_n}{2}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$

$$i_{dmod} = g_m v_{gs} + v_{ds}/r_o$$

$$g_m = \frac{dI_{Dmod}}{dV_{GS}} = k_n(V_{GS}-V_{TH})(1+\lambda V_{DS}) = \frac{2I_{modD}}{V_{GS}-V_{TH}}$$

$r_o = \frac{1}{\frac{dI_{Dmod}}{dV_{DS}}} = \frac{1}{\lambda I_{Dmod}} = \frac{V_A}{I_{Dmod}}$ where V_A is from data sheet

MODEL: (Triode) Without channel modulation: $I_D = \frac{1}{2}k_n[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2]$

$$i_d = g_m v_{gs} + v_{ds}/r_o$$

$$g_m = \frac{dI_D}{dV_{GS}} = k_n V_{DS}$$

$r_o = \frac{1}{\frac{dI_D}{dV_{DS}}} = \frac{1}{k_n[(V_{GS}-V_{TH})-V_{DS}]}$ from I_D, not λ

With channel modulation: $I_{Dmod} = \frac{1}{2}k_n[2(V_{GS}-V_{TH})V_{DS} - V_{DS}^2](1+\lambda V_{DS})$

$$i_{dmod} = g_m v_{gs} + v_{ds}/r_o$$

$$g_m = \frac{dI_{Dmod}}{dV_{GS}} = k_n V_{DS}(1+\lambda V_{DS})$$

$r_o = \frac{1}{\frac{dI_{Dmod}}{dV_{DS}}} = \frac{1}{k_n(V_{GS}-V_{TH})-V_{DS})(1+\lambda V_{DS})+\lambda I_{Dmod}}$ from I_D and λ

MODEL: (Linear) Without channel modulation: $I_D \approx k_n(V_{GS} - V_{TH})V_{DS}$

$$i_{dmod} = g_m v_{gs} + v_{ds}/r_o$$

$$g_m = \frac{dI_D}{dV_{GS}} = k_n V_{DS}$$

$$r_o = \frac{1}{\frac{dI_D}{dV_{DS}}} = \frac{1}{k_n(V_{GS} - V_{TH})} \qquad \text{from } I_D\text{, not } \lambda$$

With channel modulation: $I_{Dmod} = k_n(V_{GS} - V_{TH})V_{DS}(1 + \lambda V_{DS})$

$$i_{dmod} = g_m v_{gs} + v_{ds}/r_o$$

$$g_m = \frac{dI_{Dmod}}{dV_{GS}} = k_n V_{DS}(1 + \lambda V_{DS})$$

$$r_o = \frac{1}{\frac{dI_{Dmod}}{dV_{DS}}} = \frac{1}{k_n(V_{GS} - V_{TH})(1 + 2\lambda V_{DS})} \qquad \text{from } I_D \text{ and } \lambda$$

MODEL: (Linear) Without channel modulation: $I_D \approx k_n(V_{GS} - V_{TH})V_{DS}$

$$i_d = k_n V_{DS} v_{gs} + k_n(V_{GS} - V_{TH}) v_{ds}$$

$$g_m = \frac{\partial i_d}{\partial v_{gs}} = k_n V_{DS}$$

$$r_o = \frac{1}{\frac{\partial i_d}{\partial v_{ds}}} = \frac{1}{k_n(V_{GS} - V_{TH})}$$

With channel modulation: $I_{Dmod} = k_n(V_{GS} - V_{TH})V_{DS}(1 + \lambda V_{DS})$

$$i_d = k_n V_{DS}(1 + \lambda V_{DS}) v_{gs} + [k_n(V_{GS} - V_{TH})(1 + \lambda V_{DS}) + \lambda I_D] v_{ds}$$

$$g_m = \frac{\partial i_{dmod}}{\partial v_{gs}} = k_n V_{DS}(1 + \lambda V_{DS})$$

$$r_o = \frac{1}{\frac{\partial i_d}{\partial v_{ds}}} = \frac{1}{k_n(V_{GS} - V_{TH})(1 + \lambda V_{DS}) + \lambda I_D}$$

REF:
[1] Adel S. Sedra and Kenneth C. Smith, *Microelectronic Circuits, 5th Ed.,* New York, NY: Oxford University Press, 2004.

[2] Behzad Razavi, *Fundamentals of Microelectronics with Robotics and Bioengineering Applications, 3rd Ed., Enhanced eText,* New York, NY: Wiley, 2021. ISBN: 9781119694397

[3] Becker (https://electronics.stackexchange.com/users/309451/becker), Does channel length modulation take effect in triode region of MOSFET?, URL (version: 2024-10-09): https://electronics.stackexchange.com/q/727466

[4] Hearth (https://electronics.stackexchange.com/users/57556/hearth), Does channel length modulation take effect in triode region of MOSFET?, URL (version: 2024-10-09): https://electronics.stackexchange.com/q/727468