TOOL: For the NMOS FET circuit configuration shown in Fig. 1, the DC operating point for the FET operating in saturation with no channel modulation is found using two equations relating V_{GS} to I_D.

Solving the two equations gives the values of for V_{GS} and thus I_D, allowing other currents and voltages to be found. The following solution for V_{GS}, is derived below.

$$V_{GS} = V_k \left(-1 + \sqrt{1 + \frac{2V_\Delta}{V_k}} \right) + V_{TH}$$

where

$$V_k \equiv \frac{1}{k_n R_S}$$

$k_n \equiv$ transconductance of FET in $\mathrm{A/V^2}$

$V_\Delta \equiv V_G - \mathrm{V_{SS}} - V_{TH}$

$V_{TH} \equiv$ Threshold of FET in V

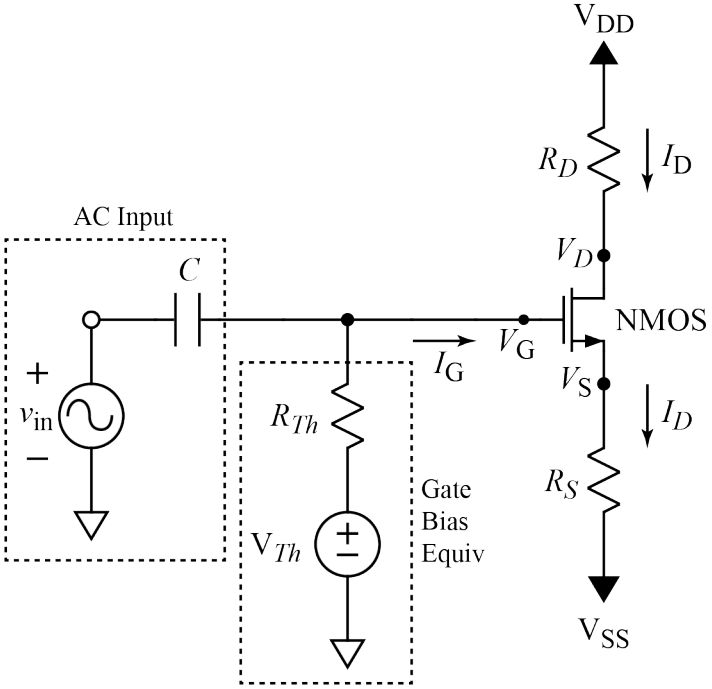


Fig. 1. NMOS FET in CS amplifier.

DERIV: We assume the value of DC gate voltage, V_G, has been found. The typical approach is to find the Thévenin equivalent voltage of the circuitry attached to the gate, as shown in Fig. 1. Note that we assume the gate draws no current, so there is no voltage drop across R_{Th}, and thus $V_G = V_{Th}$. Also, any AC input signals coupled through a series capacitor will be effectively disconnected at DC owing to the infinite impedance of the capacitor at DC.

The first equation in the solution is the standard equation for FET drain current in saturation:

$$I_D = \frac{k_n}{2}(V_{GS} - V_{TH})^2. \tag{1}$$

where

$k_n \equiv$ transconductance of FET in $\mathrm{A/V^2}$

$V_{TH} \equiv$ Threshold of FET in V

The second equation is an expression of V_{GS} in terms of V_G and V_S where V_S is expressed in terms of I_D and R_S.

$$V_{GS} = V_G - V_S = V_G - (\mathrm{V_{SS}} + I_D R_S). \tag{2}$$

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$$V_{GS} = V_G - V_S = V_G - (\mathrm{V_{SS}} + I_D R_S). \tag{3}$$

Solving (2) for I_D in terms of V_{GS} yields a linear equation.

$$I_D = \frac{V_G - \mathrm{V_{SS}} - V_{GS}}{R_S}. \tag{4}$$

We now equate (2) and (4) to get one equation in V_{GS}. (Note that, although we could also solve (3) for V_{GS} and substitute in (2), we would get a quadratic equation for I_D with two solutions, and it is less obvious which solution is valid as compared to using the two values of V_{GS} we obtain with the recommended approach.)

Equating (2) and (4) gives our desired quadratic equation:

$$\frac{k_n}{2}(V_{GS}-V_{TH})^2=\frac{V_G-V_{SS}-V_{GS}}{R_S} \tag{5}$$

Rather than expanding the squared term, it is convenient to define a new variable representing $V_{GS}-V_{TH}$:

$$V_{OV}\equiv V_{GS}-V_{TH}. \tag{6}$$

Using this new term, the squared term is simpler.

$$\frac{k_n}{2}V_{OV}^2=\frac{V_G-\mathrm{V_{SS}}-(V_{OV}+V_{TH})}{R_S}. \tag{7}$$

Now some algebra.

$$\frac{k_n}{2}V_{OV}^2+\frac{V_{OV}}{R_S}-\frac{V_G-\mathrm{V_{SS}}-V_{TH}}{R_S}=0\,\mathrm{A}. \tag{8}$$

or

$$V_{OV}^2+\frac{2}{k_nR_S}V_{OV}-\frac{2}{k_nR_S}(V_G-\mathrm{V_{SS}}-V_{TH})=0\,\mathrm{V}^2. \tag{9}$$

Define constants to simplify the notation.

$$V_k\equiv\frac{1}{k_nR_S} \tag{10}$$

$$V_\Delta\equiv V_G-\mathrm{V_{SS}}-V_{TH} \tag{11}$$

In the new notation, our quadratic is

$$V_{OV}^2+2V_kV_{OV}-2V_kV_\Delta=0\,\mathrm{V}^2. \tag{12}$$

The solution of the quadratic uses the simplified version of the quadratic formula that applies when the coefficient of the squared term is unity.

$$V_{OV}=-V_k\pm\sqrt{V_k^2+2V_kV_\Delta}. \tag{13}$$

Since V_{OV} > \SI{0}{V} for saturation, we use the positive root.

$$V_{OV}=-V_k+\sqrt{V_k^2+2V_kV_\Delta}=V_k\left(-1+\sqrt{1+\frac{2V_\Delta}{V_k}}\right). \tag{14}$$

or

$$V_{GS}=V_k\left(-1+\sqrt{1+\frac{2V_\Delta}{V_k}}\right)+V_{TH}. \tag{15}$$