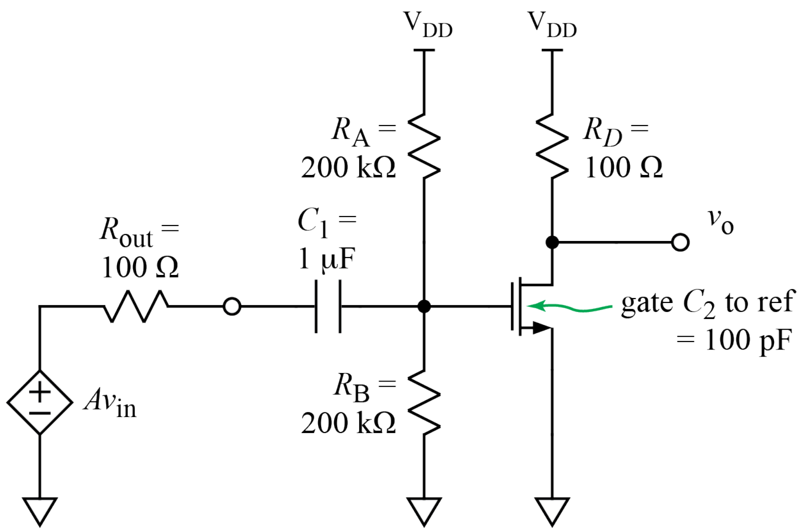
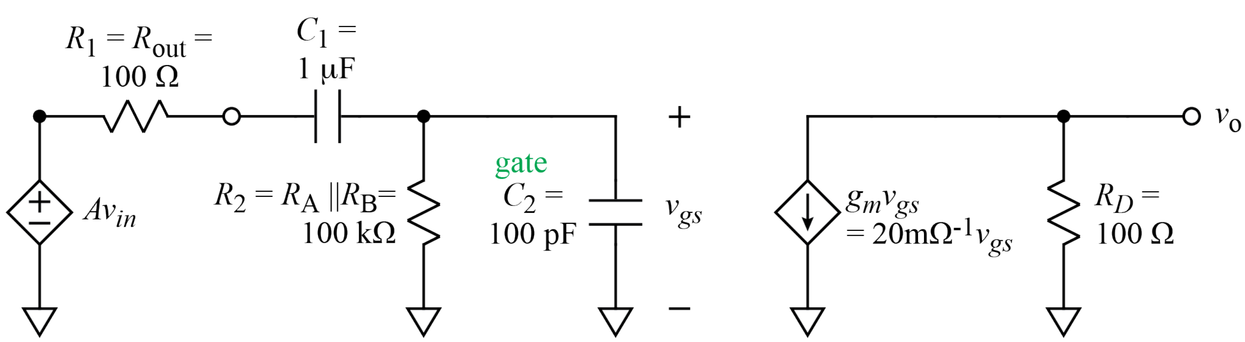
Ex: When capacitances in a transistor circuit are included in the AC model, the output of the circuit becomes frequency dependent. The circuit below shows a case where the gate capacitance of an FET is taken into account, causing a high-frequency rolloff of gain to occur. The AC coupling capacitor, C_1, also causes very low frequencies to be attenuated. Thus, this circuit has a flat passband with cutoff frequencies on the low and high ends.



The AC model of the circuit is shown below and includes the gate capacitance and transconductance value of the FET.



a) Find the tansfer function, $H(s)$, for the circuit gain from input Av_{in} to v_o:

$$H(s) = \frac{v_o}{Av_{in}}.$$

b) Make a magnitude Bode plot for the transfer function on the graph below.

c) Make a phase shift Bode plot for the transfer function on the graph below.

SOLN: a) Define $R_1 \equiv R_{\text{out}}$ and $R_2 \equiv R_A || R_B$.

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2 || \frac{1}{sC_2}}{R_1 + \frac{1}{sC_1} + R_2 || \frac{1}{sC_2}} g_m R_D$$

$$R_2 || \frac{1}{sC_2} = \frac{1}{sC_2} \cdot sR_2C_2 || 1 = \frac{R_2}{sR_2C_2 + 1}$$

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{\frac{R_2}{sR_2C_2 + 1}}{R_1 + \frac{1}{sC_1} + \frac{R_2}{sR_2C_2 + 1}} g_m R_D$$

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R_2}{\left(R_1 + \frac{1}{sC_1}\right)(sR_2C_2 + 1) + R_2} g_m R_D$$

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{sR_2C_1}{(sR_1C_1 + 1)(sR_2C_2 + 1) + sR_2C_1} g_m R_D$$

SOLN: b) Define terms:

$\tau_{11} \equiv R_1C_1 = 100\ \mu\text{s}$ $\tau_{22} \equiv R_2C_2 = 10\ \mu\text{s}$ $\tau_{21} \equiv R_2C_1 = 100\ \text{ms}$

for $R_1 = 100\ \Omega$, $R_2 = 100\ \text{k}\Omega$, $C_1 = 1\ \mu\text{F}$, and $C_2 = 100\ \text{pF}$

$$H(s) = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{s\tau_{21}}{(s\tau_{11} + 1)(s\tau_{22} + 1) + s\tau_{21}} g_m R_D$$

$$(s\tau_{11} + 1)(s\tau_{22} + 1) + s\tau_{21} = \tau_{11}\tau_{22}s^2 + (\tau_{11} + \tau_{22} + \tau_{21})s + 1$$

$$\tau_{11}\tau_{22}s^2 + (\tau_{11} + \tau_{22} + \tau_{21})s + 1 = as^2 + bs + 1 = \left(\frac{s}{\omega_1} + 1\right)\left(\frac{s}{\omega_2} + 1\right)$$

$$\frac{1}{\omega_1\omega_2} = \tau_{11}\tau_{22} = a \;\; \omega_1\omega_2 = \frac{1}{a} \qquad \omega_2 = \frac{1}{a\omega_1}$$

$$\frac{1}{\omega_1} + \frac{1}{\omega_2} = \tau_{11} + \tau_{22} + \tau_{21} = b \qquad \omega_1 + \omega_2 = b\omega_1\omega_2 \qquad \omega_1 + \frac{1}{a\omega_1} = \frac{b}{a}$$

$$\omega_1^2 + \frac{1}{a} = \frac{b\omega_1}{a} \qquad \omega_1^2 - \frac{b\omega_1}{a} + \frac{1}{a} = 0$$

where

$$\omega_1 = \frac{b}{2a}\left(1 - \sqrt{1 - 4a/b^2}\right) \qquad \omega_2 = \frac{b}{2a}\left(1 + \sqrt{1 - 4a/b^2}\right)$$

$\sqrt{1-x} \approx 1 - x/2$ for x small

Here, we have

$$x = \frac{4a}{b^2} = \frac{4\tau_{11}\tau_{22}}{(\tau_{11}+\tau_{22}+\tau_{21})^2} = \frac{100\cdot 10}{(100+10+100\text{k})^2} \approx \frac{4\text{k}}{(100\text{k})^2} = \frac{4}{10\text{M}}$$

for

$$\tau_{11} \equiv R_1C_1 = 100\ \mu\text{s} \qquad \tau_{22} \equiv R_2C_2 = 10\ \mu\text{s} \qquad \tau_{21} \equiv R_2C_1 = 100\ \text{k}\mu\text{s}$$

$$\omega_1 \approx \frac{b}{2a}\left(1 - [1 - 2a/b^2]\right) = \frac{b}{2a}\left(2a/b^2\right) = \frac{1}{b}$$

$$\omega_2 \approx \frac{b}{2a}\left(1 + [1 - 2a/b^2]\right) = \frac{b}{2a}(2) = \frac{b}{a}$$

Here τ_{21} dominates over τ_{11} and τ_{22}.

b is approximately τ_{21}, and a is τ_{11} times τ_{22}

So our break frequencies are about $1/\tau_{21}$, which is small, and $\tau_{21}/\tau_{11}\tau_{22}$, which is large.

For $\tau_{11} = 100\ \mu\text{s}$, $\tau_{22} = 10\ \mu\text{s}$, and $\tau_{21} = 100\ \text{ms} = 0.1$, $a = 1\ \text{ns}^2$ and $b = 100\ \text{ms}$

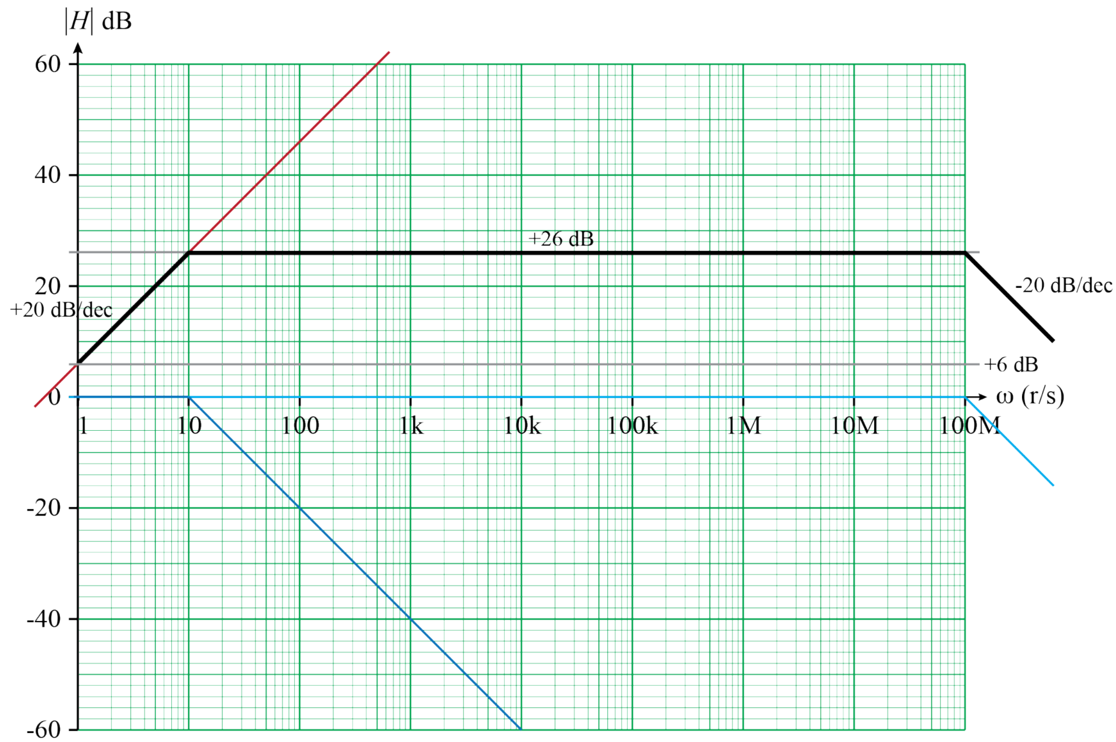
$$\omega_1 = \frac{1}{100\ \text{ms}} = 10\ \text{r/s}$$

$$\omega_2 = \frac{100\ \text{ms}}{1\ \text{ns}^2} = 100\ \text{Mr/s}$$

Bode magnitude plot vertical offset is $\tau_{21}\, g_m R_D = 100\ \text{ms} \cdot 2 = 2\ \text{s} \Rightarrow 6$ dB at 1 r/s.

$$H(s)=\frac{\mathbf{V}_o}{\mathbf{V}_i}=2\frac{s}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100\,\mathrm{M}}+1\right)}$$

Magnitude plot rises at 20 dB/dec until ω_1 = 10 r/s, is flat at 26 dB until ω_2 = 100 Mr/s, and then falls at −20 dB/dec.



Soln: c) Bode phase plot starts at 90° from numerator, falls 90° from $\omega_1/10$ to $10\omega_1$ where it stays at 0° until falling 90° from $\omega_2/10$ to $10\omega_2$. After $10\omega_2$, it stays at −90°.