TOOL: A hyper-plane is defined by a dot-product formula:

$$y = a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_N x_N = \vec{a} \circ \vec{x}_+$$

where

$$\vec{x}_+ \equiv (1, x_1, x_2, \ldots, x_N) = (1, \vec{x})$$

The following matrix equation is solved by the vector, $\vec{a}$, that defines the hyper-plane passing through $N + 1$ specified points, $(x_1, y_1), \ldots, (x_N, y_N)$:

$$\begin{bmatrix} 1 & x_{11} & \cdots & x_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_{N+1,1} & \cdots & x_{N+1,N} \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_{N+1} \end{bmatrix}$$

or in matrix notation:

$$X_+ \vec{a} = \vec{y}$$

Assuming the X_+ matrix is full rank, the solution of the above equation yields $\vec{a}$:

$$\vec{a} = X_+^{-1} \vec{y}$$