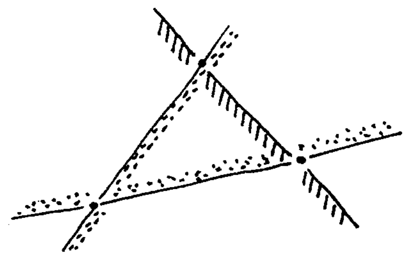
Neil E Cotter
1? ?y 1994

tool: To find the triangle containing a data point, test whether the data point lies on the interior side of each side of ~~the~~ a triangle. Search through all the triangles until you find the correct one.

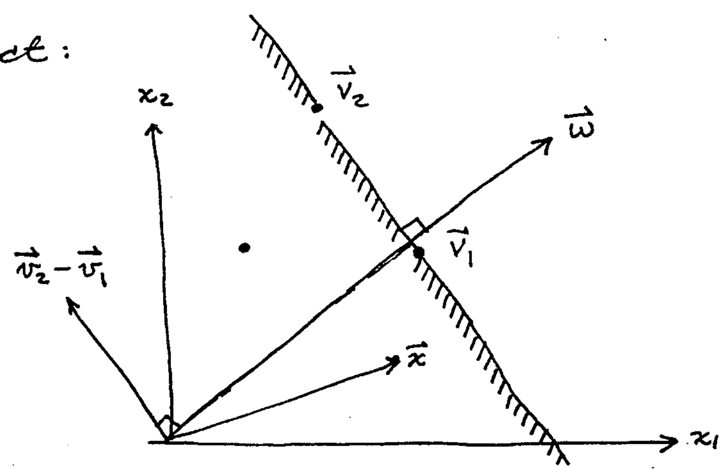
pict:



shading indicates interior sides

tool: A dot product of $\vec{x}$ with a vector $\vec{w}$ perpendicular to the side of a triangle determines whether $\vec{x}$ lies on the interior or exterior of the side.

pict:



We can find $\vec{w}$ such that $\vec{w} \perp (\vec{v}_2 - \vec{v}_1)$ where $\vec{v}_2 - \vec{v}_1$ is parallel to side defined by $\vec{v}_1, \vec{v}_2$. Points $\vec{x}$ lying on this $\vec{v}_1, \vec{v}_2$ side satisfy the following eq'n:

$$\vec{w} \circ \vec{x} = c \text{ (constant)}$$

We can scale $\vec{w}$ to make $c = 1$ to get:

$\vec{x}$ on boundary or side when $\vec{w} \circ \vec{x} = 1$ $\qquad$ $\vec{w} \perp$ side

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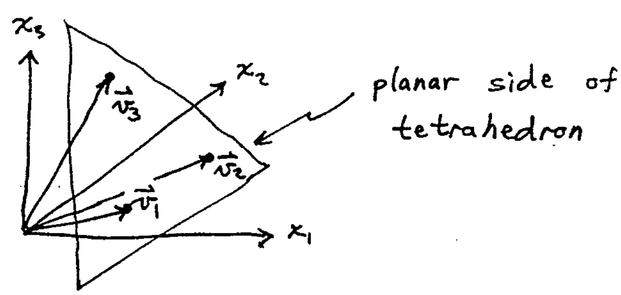
tool (cont): If we have $\vec{\omega} \circ \vec{x} > 1$ we are on one side of the side. If we have $\vec{\omega} \circ \vec{x} < 1$ we are on the other side of the side.

comment: We can find the $\vec{w}$'s for the sides and store them ahead of time. Then all we need to do is compute $\vec{w} \circ \vec{x}$ when we are evaluating $f(\vec{x})$.

note: In 3 dimensions we can still use $\vec{w} \circ \vec{x} = 1$ to define which $\vec{x}$ are on a side of a tetrahedron (a 3-dim triangle).

tool: To find $\vec{w}$ perpendicular to a planar side of a tetrahedron, we must find a vector $\perp$ to $\vec{v}_1 - \vec{v}_2$ and $\vec{v}_2 - \vec{v}_3$ where $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are the three data points defining the side of the tetrahedron.

pict:



Note that $\vec{v}_1 - \vec{v}_2$ and $\vec{v}_2 - \vec{v}_3$ are vectors parallel to the planar side. To find $\vec{w}$ we solve the following equations

$$\begin{aligned} \vec{\omega} \circ \vec{v}_1 &= 1 \\ \vec{\omega} \circ \vec{v}_2 &= 1 \\ \vec{\omega} \circ \vec{v}_3 &= 1 \end{aligned} \quad \text{or} \quad \begin{bmatrix} - \vec{v}_1 - \\ - \vec{v}_2 - \\ - \vec{v}_3 - \end{bmatrix} \begin{bmatrix} | \\ \vec{\omega} \\ | \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{or} \quad V \circ \vec{w} = \vec{1}$$

Thus, $\vec{w} = V^{-1} \cdot \vec{1}$.