EE 5961-2/6961-2 Lecture #27 (cont)

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Daubechies's Regular Filters and Wavelets

Perfect reconstruction and orthogonal if

$$|M_0(e^{j\omega})|^2 + |M_0(e^{j(\omega+2\pi)})|^2 = 1 \qquad (1)$$

where $M_0(1) = 1$, $M_0(\pi) = 0$ assumed

For regularity $M_0(e^{j\omega}) = \left[\frac{1}{2}(1+e^{j\omega})\right]^N R(e^{j\omega})$ $N \geq 1$, $R(1)=1$

def $y = \cos^2\frac{\omega}{2}$ $\quad P(1-y) = |R(e^{j\omega})|^2$

Then $y^N P(1-y) + (1-y)^N P(y) = 1$ for (1), $P(y) \geq 0$ $y \in [0,1]$

suppose $\sup_{\omega} |R(e^{j\omega})| = \sup_{y\in[0,1]} |P(y)|^{1/2} < 2^{N-1}$

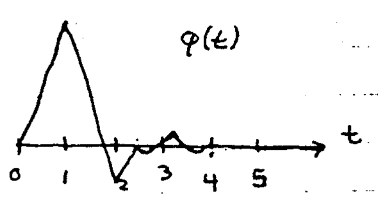
Then $M_0(\omega)$ converges in $\Phi(\omega) = \prod_{k=1}^{\infty} M_0\left(\frac{\omega}{2^k}\right)$

soln: $P(y) = \sum_{j=0}^{N-1} \binom{N-1+j}{j} y^j + y^N Q(y)$

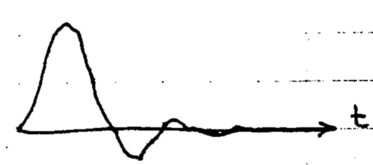
where Q is antisymmetric polynomial (i.e. odd?)

Can solve for R from P. Can choose zeros inside unit circle for min phase soln.

ex: $N=3$



$N=4$



(strange wiggles & bumps)

ref: Wavelets and SubBand Coding by Martin Vetterli and Jelena Kovačević, Prentice Hall PTR, Upper Saddle River, NJ, 1995 ISBN 0-13-097080-8.