Wavelets

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def: multiresolution analysis ≡ sequence of embedded, closed, function subspaces satisfying following requirements:

$$\ldots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \ldots \quad \text{(embedded subspaces)}$$

i) $\overline{\bigcup_{m\in\mathbb{Z}}} V_m = L_2(\mathbb{R})$ (completeness)

i.e. the closure of the union of all the subspaces = the set of all continuous, real-valued, finite-energy $\left(\int_{-\infty}^{\infty} f^2(t)\,dt < \infty\right)$ functions on the real line

Note: Since each subspace is embedded in the next, we could also say $\overline{\lim_{m\to-\infty} V_m} = L_2(\mathbb{R})$.

Also, we may think of the standard union $\cup$ operation instead of $\overline{\cup}$ without losing anything except functions that are the limit of a sequence of functions.

Note:

orthonormal if

$$\sum_{k=-\infty}^{\infty} \left|\Phi(\omega + 2k\pi)\right|^2 = 1$$

for all ω

$\Phi(\omega) = \mathcal{F}\{\varphi(t)\}$

↑ scaling func